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TRANSLATION OF

THE MECHANICAL PROPERTIES OF ICE

(Mekhanicheskie svoistva l'da)

by

K. F. Voitkovskii

Translated
by

The American Meteorological Society
AF Contract 19(604)-6113 and
The Arctic Institute of North America
AF Contract 19(604)-8343

for Terrestrial Sciences Laboratories,
Air Force Cambridge Research Laboratories,
Bedford, Massachusetts.

Voitkovskii, K. F. Mekhanicheskie svoistva l'da, Moscow, Izd. Akademii Nauk SSSR, 1960. 100 pp.

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Research carried out at the Severo-Vostochno Otdelenie Instituta Merzlotovedeniia imeni V. A. Obrucheva (North-eastern Division of the V. A. Obruchev Permafrost Institute).

PREFACE

Ice is one of the most widely distributed solids on the earth's surface encountered by man in everday life. We know that ice sometimes inflicts great losses on the national economy; it obstructs navigation, exerts an adverse influence on port installations, bridge supports, hydrotechnical equipment, et al. However, ice is also widely employed as a construction material for ice storehouses, ice causeways and the like and for ice crossings and ice roads. Further, it is used to combat aridity.

All this indicates the need for sound knowledge of the physical and mechanical properties of ice. Studies of these properties have been made and extensive data have appeared in the special literature, but it is very difficult to make practical use of this information because the quantitative parameters of the mechanical properties of ice show very large discrepancies. For example, the ultimate compressive strength of ice found by testing ice samples varies from 10 to 100 kg/cm^2 and more, i.e., it may vary by a factor of ten or more. The data on the plastic properties of ice reveal even greater discrepancies. In view of this, the need has arisen to analyze and generalize all these data, to attempt to explain the reasons for the large discrepancies in the various parameters, to establish the laws of their change and to find the most reliable characteristics of the mechanical properties of ice in order to make recommendations for engineering practice. The present work is devoted to the solution of these problems. It is based on a generalization of the data in the literature on the properties of ice and on the experimental work of the author on studies of the plastic properties of ice carried out at the V. A. Obruchev Permafrost Institute of the Academy of Sciences of the USSR during the period 1954 through 1958. The author wishes to express his deep gratitude to Corresponding Member of the Academy, N. A. Tsytovich, to Professor S. S. Vialov and to Professor B. A. Savel'ev for a number of valuable suggestions which have been considered in the present work.

CHAPTER I

THE STRUCTURE AND THE PHYSICAL PROPERTIES OF ICE

Ice has crystalline structure. Ice crystals are optically uniaxial and belong to the hexagonal system. The external form of the crystal varies and depends on the conditions of the formation and growth of the crystal. However, one may distinguish three basic types of ice crystals: tabular, columnar and needle (acicular). The crystal size varies greatly (from a fraction of a millimeter to one meter and more) and changes continually because of recrystallization processes, by which some crystals grow at the expense of others.

Ice crystals have a sharply defined mechanical anisotropy, as a function of the direction in which forces act with respect to the basal plane (the plane perpendicular to the optic axis of the crystals). The atoms in the space lattice of the ice crystal are arranged such that a disturbance in the basal plane breaks only two atomic bonds per unit cell, while a disturbance in any plane perpendicular to the basal plane requires the breaking of at least four bonds per cell (Owston and Lonsdale, 1948). Therefore, the structure of an ice crystal may be represented as a collection of numerous, very thin, durable but flexible plates (McConnel, 1891). The intervals between the elementary plates (the planes of closest packing of atoms) are planes of weakness, along which relative slipping of the plates may occur.

Due to the anisotropy of the ice properties, one must consider the structure of ice and the direction of the optic axes of its crystals.

In nature one encounters various types of ice, distinguished by structure, type of distribution and other properties. Actually, large single crystals of ice are rarely found. For the most part, one finds polycrystalline ice, which consists either of randomly oriented fused crystals (granular ice) or intergrown individual crystals, whose axes are approximately parallel.

Ice structure is a function of its mode of formation. The following basic types may be distinguished (Tsytovich and Sumgin, 1937):

a) Continuous crystalline, which forms during the calm freezing of water;

b) Needle, often with air bubbles, which forms at the point of contact of water and ice;

c) Lamellar, which forms during the periodic freezing of individual layers of water or during the densification of individual layers of wet snow;

d) Firn or granular, which forms during the freezing of snow;

e) Fine-aggregate irregular, which forms during alternate freezing and mixing (observed in the upper ice layer of large reservoirs);

f) Loose-flaky, observed in a newly fallen snow cover and also during the freezing of water which condenses from vapor.

We now have a more complete and better genetic classification of ice developed by P. A. Shumskii (Shumskii, 1955), in which all types of fresh ice are taken into account and a detailed description is given of the conditions of their formation and mode of occurrence, their structure, crystal orientation and the nature of their air inclusions. The broad scope of the Shumskii classification makes it somewhat cumbersome, however, and since we are not concerned with the genesis of ice, we feel we can limit ourselves to the above simplified classification and refer our readers to Shumskii's work for a more complete classification.

When water freezes calmly, crystals with optic axes parallel to the surface of freezing predominate in the upper layer of the ice cover, while crystals with vertical axes predominate in the lower layers. In lake ice, according to B. A. Savel'ev (1953), crystals with optic axes parallel to the surface of freezing are encountered approximately to a depth of 18 cm, while below that all crystals have optic axes perpendicular to the plane of freezing. With increasing depth from the upper surface, some of the crystals wedge out and the lateral dimension of the remaining crystals increases.

In the case of turbulent freezing of water, the axes of the ice crystals have random orientation.

The melting of ice and sublimation (transition to the vapor state) are functions of temperature and pressure. Under specific temperatures

and pressures, ice and water or ice and vapor may enter into equilibrium with each other. The curves of the equilibrium of these phases (fig. 1) show the limits of the stable state of the ice. The point of intersection of the indicated curves, the so-called triple point, in which the system ice-water-vapor is in equilibrium corresponds to a temperature of + 0.0099°C and a pressure of 0.006 atmosphere. The melting point of pure ice at normal atmospheric pressure is 0°C. The melting point is reduced as hydrostatic pressure is increased. It has been established that a pressure increase of 1 kg/cm^2 corresponds to a 0.0075°C reduction of the melting point. Oriented unilateral pressure can also cause some change in the melting point, but it will be quite negligible, less than 0.01°C (Shumskii, 1955).

Water may freeze at the same temperatures (as a function of pressure) at which ice melts only at the boundary of an extant crystal phase. Therefore, when there are no centers of crystallization, water may be "supercooled" considerably below the temperature at which thermodynamic conditions are created for converting water into ice. Cases are known where water droplets have been supercooled to -72°C. Under natural conditions, there are always foreign particles in water which become centers of crystallization, therefore, ordinarily water cannot be supercooled more than a few degrees.

In addition to ordinary ice, various polymorphour modifications of ice are known. However, they exist only under great pressures (from 2,000-50,000 atm) and are not encountered under normal conditions, therefore, we shall not investigate these modifications of ice in the present work.

Impurities in ice. In ice one usually finds a certain number of impurities, inclusions of air or of gases and salts. Furthermore, under specific conditions (heat influx, increased pressure) ice also contains water. In natural ice one may find various solid inclusions, e.g., insoluble mineral fragments.

The gaseous inclusions in ice come either directly from the atmosphere or from freezing water. The chemical composition of these inclusions is usually close to that of atmospheric air. The air inclusions

in ice are usually tiny spherical voids or elongated closed cells. Open pores and cracks are also found. Dense, transparent ice either contains no macropores at all or very few of them (porosity within the limit of 1 cm^3/kg). If there is a large quantity of air inclusions in the ice, the ice is less transparent, i.e., cloudy. Such ice is usually found in the middle and the lower layers of the ice cover. In this case, the porosity of the ice generally varies from 1 to 50 cm^3/kg. Ice with a very large number of air pores (from 50-400 cm^3/kg) also occurs. Such ice is opaque and looks like snow (Savel'ev, 1953). Usually it forms as a result of the freezing together of moistened snow.

The presence of dissolved salts in water changes the conditions of freezing of the water. Upon cooling and passing into the solid state, the solution becomes in-homogeneous and breaks up into its components. When a low-concentration solution is cooled below the freezing point of pure water, pure ice begins to form from it and the solution concentration increases. When a high-concentration solution is cooled, it becomes supersaturated and salt crystals begin to form from it.

Thus, for each solution temperature below the freezing point of pure water, there are two maximum equilibrium concentrations of salts, beyond which ice or salt crystals separate out (fig. 2). The temperature corresponding to the point of intersection of these limits is called the eutectic temperature. At this temperature and the corresponding solution concentration, both components separate out simultaneously and the solution composition remains unchanged during this freezing period, i.e., the solution freezes completely. As a result, a eutectic mixture of ice and salt crystals forms. If the initial concentration of the solution was less than the eutectic, after cooling below the eutectic temperature a mixture of eutectic and ice forms, called the hypoeutectic, but if the initial concentration of the solution is greater than the eutectic, a mixture of eutectic and salt forms, called the hypereutectic. Thus, salts in the ice above the eutectic temperature are in the form of a liquid brine, and its concentration increases and the quantity decreases with intensification of freezing. When the salt content is small, almost all the brine in the ice is concentrated in the form of films or isolated inclusions at the crystal boundaries. An increase in the salt content leads to the formation of inter-

layers of brine within the crystals in the basal planes, separating the crystals into a number of plates (Savel'ev, 1953; Shumskii, 1955).

The presence of films, isolated inclusions and especially interlayers of liquid brine exerts a considerable influence on the mechanical properties of ice, reducing the ice strength. However, it should be noted that the amount of liquid brine in fresh-water ice is usually negligible and can exert some influence on the mechanical properties of ice only at a temperature close to the melting point.

Regelation and recrystallization. Ice can regel or freeze together. This property consists in the following: external forces may cause some melting of particles at the points of contact of ice particles or pieces of ice. The water which forms in this case is extruded to places where pressure is lower and freezes there, as a result ice particles freeze together. The freezing together of ice surfaces occurs more slowly and can take place without any pressure and without the participation of a liquid phase, as a result of the sublimation* of ice and recrystallization. Consequently, hairline cracks in ice cannot exist for a long period of time.

Recrystallization takes place continuously in ice and is manifested in the spatial displacement of the boundary between crystals, in the change of size, shape and total number of crystals and in the change of crystal orientation. The rounding of sharp edges and corners is observed in individual crystals. Crystals strive toward the equilibrium form (a sphere), which is characterized by a minimum free energy. In polycrystalline ice, the principle of minimum free energy is manifested in the tendency toward the fusion of crystals and reduction of the number of crystals. The larger crystals grow because of the reduction of volume (and even the complete disappearance) of the smaller crystals, i.e., a "selective" recrystallization occurs (Shumskii, 1955). Rounding is rapid only when angular crystals are present, while selective recrystallization takes place only when very small crystals are present. These processes diminish as the crystals become rounded and the tiny crystals disappear.

* By sublimation we mean the process of the distillation of ice from one place to another through the vapor state, i.e., volatilization, the migration of vapor and its crystallization.

The crystallization processes are more intensive in ice in a state of stress under the influence of various mechanical forces. In this case, due to the mechanical anisotropy of the crystals, a non-equilibrium stressed state is created and recrystallization begins with the growth of the less stressed crystals or their parts at the expense of the more heavily stressed crystals. The orientation of the crystals may change in this case. Crystals with basal planes close to the directions of shear experience a smaller stress during the deformation process than crystals oriented differently and under heavier stress and they grow at the expense of the more heavily stressed crystals.

Recrystallization consists in the transfer of molecules from the space lattice of one crystal to the space lattice of another crystal adjacent to it. Recrystallization may also take place by the redistribution of material through the vapor or liquid phase. These latter types of recrystallization play a considerable role at temperatures close to zero.

Density and specific volume. The density of pure ice at 0°C and a pressure of 1 atmosphere is 0.9168 g/cm^3, while the specific volume is 1.0908 cm^3/g. The density of water under these conditions is 0.999863 g/cm^3. Water expands 9% upon freezing. When the ice contains pores and impurities, its density differs slightly from the above. The densitv of pore-free ice changes but slightly under the influence of pressure. According to the data of B. P. Veinberg (1940), the compressibility factor is approximately (1-5) x 10^{-5} per atm. Pressure may exert a substantial influence on the density of porous ice only, reducing the porosity and correspondingly increasing the density.

Thermal expansion. The coefficient of expansion of ice is a function of temperature, increasing as temperatures increase. In the temperature range -20°C to 0°C, the coefficient of linear expansion is, on an average, 5.5 x 10^{-5}, while the coefficient of volumetric expansion is correspondingly 16.5 x 10^{-5} per °C. In the temperature range -40°C to -20°C, the coefficient of linear expansion, according to the experiments of Andrews (see Veinberg, 1940) is about 3.6 x 10^{-5} per °C.

The specific heat of ice varies as the temperature, decreasing as the temperature decreases. This relationship may be expressed by the following empirical formula (Veinberg, 1940):

$$C_{ice} = 0.5057 - 0.001863\ \theta \text{ cal/g degree}$$

where θ is the absolute value of the negative temperature of ice in °C.

The latent heat of fusion of pure ice is 79.6 cal/g. The latent heat of sublimation (vaporization) of ice at 0°C is 677 cal/g.

The thermal conductivity of ice is a function of the ice temperature. The coefficient of thermal conductivity of dense ice is a function of temperature and may be expressed by the empirical formula (Veinberg, 1940):

$$\lambda_{ice} = 0.0053\ (1 + 0.0015\ \theta) \text{ cal/cm sec degree,}$$

where θ is the value of the ice temperature in °C.

The presence of air inclusions in ice reduces the coefficient of thermal conductivity of the ice. Data are now available showing that the thermal conductivity of ice crystals in the direction of the main crystallographic axis is somewhat greater than in the direction perpendicular to the axis (Shumskii, 1955). However, this difference is insignificant.

CHAPTER II

THE BASIC LAWS OF ICE DEFORMATION

When a force is applied to ice, the ice begins to deform and behaves as an elastic, plastic or brittle body depending on various factors, i.e., it deforms elastically or plastically or it experiences brittle fracture.

Some of the main characteristic properties of ice compared with other crystalline bodies are its distinctly expressed plastic properties. Under a load, ice may change form without breaking and without changing volume, like a fluid. We know, for example, that glaciers "flow" at a definite speed and, to a certain extent, such flow is reminiscent of the flow of a river. Therefore, the plastic deformation of ice is sometimes compared with the flow of a highly viscous fluid.

The area of manifestation of purely elastic properties is so small that in practice one cannot distinguish it. Usually, plastic deformations can be observed along with elastic deformations, under any stress. Elastic deformation occurs at the moment load is applied, and the plastic deformation begins immediately after the elastic. The total deformation generally consists of two components: the elastic, i.e., the reversible deformation and the plastic, i.e., the residual. In passing, it should be mentioned that plastic deformations occur only in presence of shear stresses, therefore, only elastic deformations and densification will occur under equal, hydrostatic compression of monolithic ice.

Brittle fracture of ice is observed when the stresses on the ice are increased to a certain limit, the ultimate strength of the ice,* and also, in a number of cases, under the influence of dynamic loads.

The mechanical properties of ice, i.e., the capacity of ice to resist the influences of external forces change considerably depending on temperature. The closer the temperature of the ice is to the melting point of ice, the greater are the manifestations of the plastic properties of

*) This critical stress is also called the breaking point or the limit of plasticity.

the ice and the lower its strength. This phenomenon is explained by the weakening of the cohesion of the ice molecules in the space lattice of the crystals and the possibility of rearrangement of the atoms. On the other hand, the lower the temperature is, the more difficult it is for the atoms to become rearranged in the space lattice of ice and the more apparent are the elastic and brittle properties of ice.

The structure of ice also exerts a considerable influence on the character of the ice deformation. In this connection, first let us treat the deformation of individual ice crystals.

THE DEFORMATION OF ICE MONOCRYSTALS

The character of the deformation of the monocrystal is first a function of the direction of shearing forces with respect to the basal plane of the crystal. As has already been noted in Chapter 1, an ice crystal may be represented as an accumulation of numerous, very thin strong but flexible plates perpendicular to the optic axis of the crystal. These elementary plates, about 0.06 mm thick (Nakaya, 1958), corresponding to layers of closest packing of atoms, may move relatively easily with respect to each other. During the deformation of ice, gliding is observed exclusively along the basal planes. In cases where the direction of the forces causing shear does not coincide with the basal plane, the bending and relative shearing of the elementary plates occur simultaneously. Only at a temperature close to the melting point can plastic shearing occur in any direction (Glen and Perutz, 1954), because in this case many internal bonds are broken in the crystal. Gliding may take place with approximately equal ease in any direction in the basal plane (Steinemann, 1954). This type of plastic deformation may reach any magnitude up to complete extraction of the parts of the ice crystals bounded by the basal planes.

Figure 3 shows three main possible directions of forces which cause shearing relative to the basal plane. In case 1, where the shear plane coincides with the basal plane, only translation of the elementary plates takes place and the deformation is plastic. If the shearing force acts in the direction of the main axis, i.e., if the direction of force and the

shear plane are perpendicular to the basal plane (case 2), the elementary plates of the crystal bend and small relative shearing motions of the plates occur with respect to the basal planes. After the stresses in the elementary plates reach a certain limit, the plates break. Deformation is elastoplastic and when the stresses are increased, rupture occurs. In case 3, where the direction of the shearing force coincides with the basal plane, but the shear plane is perpendicular to it, the elementary plates allow only a negligible elastic deformation. However, when the stresses are increased and when there is a corresponding increase of the elastic plane strain of the elementary plates, they may bend due to the loss of stability and some relative shearing motions in directions which do not coincide with the direction of the shearing force. A further increase of stress causes the crystal to break.

From what has been said, it is evident that two different types of deformation occur simultaneously in a monocrystal under the influence of an external force, namely: elastic deformations of the elementary plates and their relative plastic shear. These two types of deformation are closely related and exert a mutual influence on each other. We know that the elastic deformation takes place instantaneously (more exactly, at the rate of propagation of acoustic waves), while plastic deformation takes place relatively slowly. Therefore, at the moment force is applied only elastic deformation of the crystal takes place. The elementary plates in this case are still rigidly connected, as it were, while bending of the plates is difficult and the total deformation is slight. The internal shearing stresses which occur cause a corresponding relative gliding of the elementary plates and their flexure. Plastic deformation begins. The gliding of the plates and their flexure cause a redistribution of the internal stresses, which leads to a change of the rate of plastic deformation, depending on the direction of the force and the magnitude of the deformation of the crystal, the rate of plastic deformation may decrease, remain constant or increase. If the shearing stresses on the planes of weakness between the plates decreases as a result of bending and slipping of the plates, the rate of plastic deformation of a monocrystal will also decrease. However, if the flexures and the rotations of the plates takes place in a direction such that the shearing stresses between them increase, the rate of plastic deformation

will increase. In cases where there is no substantial change of the internal stresses, the deformation rate may be constant.

Figure 4 shows curves of the shearing of monocrystals of ice vs. time, with different shearing stresses, at a temperature of -2.3°C, according to the experimental data of S. Steinemann (1954). The tests were conducted such that the direction of the shearing forces coincided with the basal plane and pure shear occurred. Steinemann established that two stages of creep may be distinguished in the pure shear of a crystal. The first stage occurs during gliding when the total relative angular strain compared with the undeformed state of the crystal does not exceed 0.1-0.2; the second stage applies to large shears. In the undeformed crystal, gliding takes place relatively slowly, then the creep rate increases and a new linear segment appears. A special softening of the crystal takes place. The transition from one stage to the other is irreversible. The crystal, once deformed beyond the indicated limit for the first stage, remains soft even after several hundreds of hours. After the transitional period, the deformed state remains stable. The relationship between the strain rate $\dot{\gamma}$ and the shear stress τ for both stages is expressed by the equation $\dot{\gamma} = k\tau^n$, where n is 2.3-4.0 for the first stage and 1.3-1.8 for the second stage.

Figure 5 shows the deformation curve for the tension of an ice cylinder cut from a monocrystal of glacier ice, according to the experimental data of Jellinek and Brill (1956). The rate of deformation increases with time. Apparently this increase is explained by the above-described "softening" of the crystal and by the fact that under tension the shear stresses in the sample increase between the elementary plates, due to a reduction of the cross section of the sample and the rotation of the plates. An ice crystal permits considerable plastic tensile strain. For example, cases are known where the sample was stretched almost double its initial length and became a thin tape, but remained a monocrystal (Glen, 1952). In this case the optic axis changed direction and became almost perpendicular to the direction of strain. In those cases when the optic axis of the crystal coincided with the direction of strain and primarily elastic deformation occurred, the plastic deformation was negligible and the accelerating creep stage did not occur (Glen and Perutz, 1954).

In a crystal subjected to plastic deformation under some sort of external force, after the force has ceased to act, partial reduction of the deformation occurs with a tendency toward re-establishment of the initial form. This recovery does not take place immediately, but over a certain period of time. The recovery process is similar to that of crystal deformation in the initial period of influence of the force. At the moment the force ceases, instantaneous elastic recovery occurs due to the removal of the overall stress state; then one observes a special "reverse creep" consisting in the gradual reduction of the total deformation of the crystal (at a decreasing rate).

The work expended during the mechanical action on the crystal is converted in part into thermal energy, while the remainder is transformed into the free energy of the crystal. During plastic deformation, work is converted principally into thermal energy, due to which the temperature increases or partial melting begins. The free energy of the crystal increases during elastic deformation and also when the crystal breaks, in which case work is expended on creating an additional surface. There may also be a negligible free energy increment during plastic deformation when there is some lattice disturbance and when stresses are created which lead to hardening. The excess free energy of the crystal may be expended on processes connected with relaxation or again may be converted into mechanical work. The free energy increment may also cause recrystallization.

THE DEFORMATION OF POLYCRYSTALLINE ICE

The deformation of polycrystalline ice consists in the deformation and the relative displacement of its crystals. This second factor causes some difference between the laws of the deformation of polycrystalline and monocrystalline ice.

Polycrystalline ice in which the direction of the optic axes of most crystals coincides (a characteristic feature of the ice cover during the calm freezing of water), is characterized by mechanical anisotropy. The magnitude of the deformation of such ice as well as the deformation of individual mono crystals depends, to a considerable extent, on the direction of application

of the forces relative to the axial direction of most of the crystals. However, anisotropy is not as pronounced in polycrystals as in monocrystals, since the ice crystals may slip with respect to each other and the stressed state in the deformed ice mass is almost always irregular; thus, the shear stresses act in different directions relative to the crystal axes. However, if the ice consists of randomly oriented crystals (ice which forms during the freezing of water with mixing, during layer-by-layer freezing, during the freezing together of snow, et al.), most of it may be regarded as an isotropic body.

Let us use individual examples to illustrate the laws of deformation. When polycrystalline ice is compressed, plastic deformation or creep begins after elastic deformation. If the pressure is relatively slight (a few kg/cm^2) and the ice sample is subjected to unilateral uniform compression, a steady rate of deformation will be established (fig. 6) some time after the application of load and then the magnitude of this creep rate will be a function of pressure and temperature. In this case, the creep rate may be kept constant for a long period of time if the temperature as well as the stresses in the sample are kept constant. If a high pressure operates after the stage of steady creep is established, a stage of accelerating creep or progressive flow will begin, in which case the creep rate will increase continuously. The higher the pressure, the more rapidly will the stage of accelerating creep be established and correspondingly the stage of steady state creep will decrease. With a pressure of the order of 10 kg/cm^2 and more, the stage of steady creep practically disappears and, after some decrease of the creep rate of the sample, the stage of accelerating creep begins (fig. 7). The deformation (creep rate) of an ice sample increases with increasing pressure (Kartashkin, 1947).

It should be mentioned in passing that the stage of accelerating creep is characteristic of cases in which the examined ice sample is subjected to a constant compressive stress. Since the lateral dimension of a sample increases during longitudinal compression because of lateral expansion, in cases where the compression is produced by a static load, there will be a certain reduction of the compressive stresses in the sample, which, in turn, will reduce the compression rate.

When the pressure is increased above the ultimate strength, brittle fracture of the ice will occur.

Ice experiencing compression perpendicular to the direction of the optic axes has a somewhat greater rate of deformation and a somewhat lower critical strength than ice compressed in the direction of the crystal axes.

Tensile strains of polycrystalline ice (fig. 8) occur basically in the same manner as the described compressive strains (Kartashkin, 1947).

The difference is as follows: in the case of compression, compressive stresses act in the shear plane increasing the cohesion between the mobile particles of ice, while in the case of tension, tensile stresses act in the shear planes, reducing cohesion. As a result, conditions may arise which promote the relative shearing of the crystals and the shearing of the elementary plates in crystals, which as a whole will reduce the strength of the ice. The area cross section of an ice sample under tension decreases; it therefore considerably increases the influence of the ice structure, the inhomogeneities, and the internal weaknesses. The presence of air pockets, cracks or structural faults in an extended ice sample results in an inhomogeneous stressed state at such points and increased tensile stresses, which accelerate the deformation and increase the chances that the ice will break. Considerable plastic deformation can occur in monolithic polycrystalline ice when the tension is smooth and uniform. However, the test sample of ice may rupture even when the dynamic effect is slight or in the case of vibrations. Usually the tensile strength of ice is considerably less than the compressive strength.

Figure 9 shows several curves which characterize the laws of ice flexure. Prismatic beams, 10 x 10 x 120 cm, of ice with random structure were used for the experiments (Voitkovskii, 1956). The beams were mounted on two supports with a span of 100 cm; they sagged under their own weight and an additional load of two weights was placed symmetrically at distances of 15 cm from the center of the beam span. When the beam was loaded, at first an intensive increase of deformation was observed; then the rate of deformation gradually decreased and tended toward a constant value for the given load and temperature. The deformation may continue at this rate over a very long period of time. For example, fig. 9 shows a case where the ice beam sagged at an approximately constant rate over a period of 3000 hours, i.e , more than

four months. However, immediately after the load had been changed, the sag rate changed sharply and proved to be a function of the size of the acting load and of the load before it was changed. Thus, when the weight is removed, besides the "instantaneous" elastic decrease of sag, there is a gradual slow decrease of sag which is most pronounced during the first hour, but a new increase of sagging begins after 1-2 days, due to the weight of the sample itself.

If the stresses in the bending sample of ice exceed a certain limit, after the stage of steady creep, a stage of accelerating creep begins which results in the rupture of the sample.

Polycrystalline ice experiences considerable plastic deformation during steady bending; however, it breaks easily under a dynamic load and also in tension. The breaking of the ice by bending usually begins with shearing near the neutral axis, followed by rupture of the ice in the extended zone and by extrusion of the ice in the compression zone (Kartashkin, 1947).

Figure 10 shows experimental curves of the deformation of polycrystalline ice of random structure in pure shear with a subsequent stepwise increase of the tangential stresses.

As can be seen from the graph, immediately after the application of a shear force or after the increase of this force, there is an intensive increase of the shear strain, then the strain rate gradually decreases and approaches a value which is constant for the given conditions. The plastic deformation has no critical limit under small stresses and may take place over a very long period of time. For example, in my experiment (see fig. 15), one of the ice tubes deformed over a period of 5,000 hours under the influence of a torque which caused tangential stresses of 1 kg/cm^2 at 1°C. In this case, the strain rate was nearly constant.

If the tangential stresses exceed a certain limit, after deceleration of the strain rate during the initial period of stress and after a certain interval with an approximately constant rate of strain, the shear velocity begins to increase gradually and may finally lead to destruction of the deformable volume of ice.

In the case of a complex stressed state, the magnitude and rate of shear strain is a function not only of the tangential stresses, but also of the magnitude of the normal stresses in the shear plane. With one and the same tangential stress, the additional influence of normal stresses may change (increase) the strain rate substantially (fig. 11). In other respects, the nature of the deformation remains the same as in the case of pure shear.

Shear strains form the basis of any deformation during which the form of the body changes. In particular, any plastic deformation of ice characterized by a change in the form of the body without destruction and change of the volume results from internal relative movements of the ice particles. Therefore, the nature of the plastic deformation is approximately the same during the various modes of deformation, viz., compression, tension, bending, torsion and the complex forms. The differences lie chiefly in the nature of the destruction and the magnitude of the critical strength.

Polycrystalline ice deforms under the influence of three factors:

1) elastic and plastic deformations of the individual crystals;
2) displacements of the crystals with respect to each other;
3) destruction of the crystals.

All these factors are closely related and influence each other. In ice, recrystallization occurs simultaneously with these factors and also influences the nature of the deformation. Crystals oriented with their basal plane close to the direction of shear and which, therefore, experience less stress during the deformation process grow at the expense of the less favorably oriented, more heavily stressed crystals. The individual over-stressed crystals disintegrate and become crystal fragments which are unstressed at the beginning of their formation, grow at the expense of the older stressed crystals and then become stressed and deformed themselves. Thus, during the deformation of ice due to partial destruction of the old crystals and recrystallization, there is a partial re-establishment of the undeformed state, a special "recovery" of the ice structure. This is also explained by the fact that in a number of cases ice has practically no limits of deformation. At the moment load is applied, elastic deformation begins, caused by the exceptionally elastic deformations of the ice crystals. The elastic deformations of the crystals cause stresses within the crystals and at the contacts between crystals, resulting in plastic

deformations of the crystals, relative shearings of the crystals and, in individual cases, in destruction (disintegration). Thus, plastic deformation begins immediately after the "instantaneous" elastic deformations. At the initial moment there may be concentrations of stresses at individual points of the crystals, then internal shearing, gliding along the crystal boundaries and disintegration of individual crystals occur, which leads to some internal redistribution of stresses and to a partial balancing of the stresses. A rearrangement of particles takes place, during which the ice offers more resistance to load because it becomes more rigid, as it were, and this decreases the rate of strain.

This strain hardening is accompanied by the breaking of bonds between the elementary plates of the crystal and between the individual crystals, and by the disintegration of crystals, which leads to weakening and thus to an increased rate of strain. Thus, a load induces two opposite and simultaneous processes in polycrystalline ice; the breaking of bonds and weakening, on the one hand, and the re-establishment of bonds and strengthening, on the other. The nature of the deformation is determined by the prevalence of one or the other of these processes.

In the case of slight shear stresses following instantaneous deformation and a reduction of the strain rate during the first period after the application of load, a dynamic equilibrium is gradually established between the external forces and the total internal resistance. The increased hardness of the ice due to the increased internal deformations is compensated by the decrease of hardness due to the formation of particles of undeformed material by disintegration of the individual crystals, recrystallization and weakening. A stage of permanent plastic deformation begins, i.e., steady-state creep, which may continue for an unlimited time as long as the conditions of deformation, i.e., the temperature, the stresses and the ice structure, remain unchanged. However, usually these conditions cannot remain unchanged over a long period of time and, therefore, in practice the stage of steady state creep is limited although it may persist over a long time period. Even when the temperature and stresses are constant, the orientation of the ice crystals may change during the deformation process. Recrystallization results in the

gradual formation of a structure with main axes oriented along a line perpendicular to the shear plane (Shumskii, 1958) and this increases the strain rate.

If the shear stresses exceed a certain limit, which we shall call arbitrarily "limit of prolonged creep" (for more details, see Chapter IV), either there will be no internal equilibrium between the weakening and hardening processes or it will be short-lived. Weakening prevails over hardening and there can be no prolonged stage of steady creep. In this case, deformations develop relatively rapidly and, as a result, the bonds between the elementary plates that move with respect to each other in the ice crystals and between the individual crystals break, the internal resistance of the ice decreases. In individual cases, glide planes with weakened cohesion form in the ice, and the shear along these planes is more intense. All this gradually increases the deformation rate, which may eventually lead to the disintegration of the ice.

A change in the value of the active shear stress leads to an abrupt change of the deformation rate. If the stress increases discontinuously (with increasing load) during the deformation process, the change of the deformation rate will be similar to the change in the deformation rate described above for the initiation of stress. Following an instantaneous discontinuity of deformation, i.e., an elastic deformation corresponding to a discontinuity of stresses, deformation continues at a decreasing rate. Then, gradually a new constant rate of deformation is established corresponding to the equilibrium of the new external forces and internal resistance, or the rate of deformation increases, i.e., accelerating creep begins. A reduction of the shear stress causes a reduction of the elastic deformations of the crystals, which in turn causes plastic shearing in the crystals and their relative displacement in a direction opposite that of the initial deformation. This process combines with an increase of plastic deformations due to the acting (after reduction) stresses. The result is a quite complex type of deformation; at the moment of the reduction of stress there is a somewhat discontinuous decrease of deformation which fades in time; then a short-term stabilization of deformation begins and it increases again at a rate which increases to a value corresponding to the stress acting after the change.

As a graphic presentation, in fig. 12 we give the curve of the change in sag of an ice beam under successive loadings at various time intervals (Voitkovskii, 1957). The mechanism of the elastic aftereffect (delayed appearance of elasticity) is clearly manifested here. It consists in the following: in addition to the instantaneous elastic and irreversible plastic deformation after application of load, over a certain time interval there is a structurally reversible deformation which disappears with time after unloading. One might say this delayed, structurally reversible deformation is intermediate between elastic and plastic deformation. It consists of the following: after the load has been applied and instantaneous elastic deformation occurs, there is a gradual further increase of elastic deformation of the elementary plates of the crystals proportional to the relative plastic shearings of these plates and the individual crystals. Correspondingly, the elastic stresses in the crystals do not disappear immediately after the removal of the load, but, rather, gradually, causing plastic shearing in a direction opposite that of the initial deformation.

One manifestation of the elastic aftereffect is the relaxation of stresses in ice during its steady deformation. In this case, the elastic deformations of the crystals gradually decrease due to increasing plastic deformation and, correspondingly, the internal stresses decrease and the resistance of the ice decreases. As a result, the force needed to keep the ice sample in a given state of deformation will decrease with time. Figure 13 shows relaxation curves based on the experimental data of B. D. Kartashkin (1947). The curves show the change in force required to maintain the initial sag value of ice beams 8 x 12 cm in cross section with a span of 100 cm, as a function of time and initial load. These curves correspond to the change in stressed state of the beams and reflect the general nature of the relaxation of stresses.

The greatest reduction of stresses, relaxation, is observed immediately after the ice deformation stops increasing. Then the relaxation rate gradually decreases. The higher the initial stress, the faster the relaxation in the initial period. Since the elastic limit of polycrystalline ice is practically zero, during prolonged relaxation the stresses will also decrease to approximately zero.

The plastic deformation of polycrystalline ice is connected with the partial breaking of the internal bonds of the crystals and the bonds between crystals. Therefore, it does not always occur smoothly. K. E. Ivanov and V. V. Lavrov (1950) noted that during the bending of samples of polycrystalline ice the deformation increased discontinuously, accompanied periodically by a peculiar scraping sound. However, we did not observe discontinuities in our experiments (Voitkovskii, 1957) where considerably larger samples were used. Evidently, discontinuities of deformation can appear only during the deformation of small ice samples. The partial breaking of bonds in one crystal may have a perceptible influence on the deformation of the entire sample. In large samples of polycrystalline ice, the individual discontinuities will not be apparent in the overall deformation of the sample and the deformation will actually be very smooth due to the total effect of a large number of minute discontinuities. The author also assumes that the discontinuous nature of the deformation may be perceptible in cases where irregular shear stresses, exceeding the limit of prolonged creep at individual points, occur in the deformable ice mass. Then, at points of increased stresses, conditions may favor a progressive flow which will bring about an abrupt redistribution of the internal stresses and, possibly, a discontinuous change of the deformation rate. All these questions require further experimental verification.

Recently, P. A. Shumskii (1958) developed a new theory of the mechanism of ice straining and recrystallization based on the data of crystallographic investigations of ice, firn and snow samples which had been strained. According to his theory, six different mechanisms of ice straining can be distinguished on the basis of structure, the magnitude of the tangential and normal stresses and temperature. The first ice strain mechanism consists in slow shearing parallel to the basal planes of the crystals. In this case no structural changes of the ice are observed. The second mechanism is one whereby the mass of polycrystalline ice flows slowly under the influence of a tangential stress less than 1 kg/cm^2 and the intragranular slip along the basal planes accompanied by a slight distortion and other lattice disturbances caused by a slow migratory recrystallization of the ice and by an ordering of the structural orientation (a structure forms with an orientation of the main

axes perpendicular to the shear plane). In the third mechanism, with a high rate of flow, the intragranular slip is accompanied by curvature (distortion) and crystal lattice disturbances, by disarrangement of the structure of heavily stressed crystals and by recrystallization. The fourth mechanism is one in which a further increase of load and the strain rate with breaking and partial destruction of the crystal bonds is accompanied by intragranular slip, reduction of crystal size and the formation of random structure in the shear zone. The fifth mechanism comprises the ice straining which takes place at low pressures due to large tangential stresses (10 kg/cm^2 and more); shearing causes crack formation, slipping along the planes of fracture and disintegration of the ice. The sixth mechanism is one in which the straining occurs under great pressure and at high temperatures; there is partial internal melting of the ice due to the heat of friction, followed by freezing-together and the formation of the so-called blue bands.

This classification of the ice strain mechanisms allows one to draw a clearer picture of the physical essence of the laws of ice deformation and the specific nature of the mechanical properties of ice.

CHAPTER III

THE ELASTIC PROPERTIES OF ICE

As has already been noted, the elastic limit of ice is close to zero and ordinarily the elastic properties appear together with the plastic. Consequently, it is difficult to determine the exact value of the various factors which characterize the elastic properties of ice (the elastic modulus E, the shear modulus G and Poisson's ratio μ).

When the load acting upon the ice is changed, three different types of deformation appear: 1) elastically reversible instantaneous deformation, 2) irreversible deformation, i.e., creep and 3) the slowly reversible deformation of the aftereffect. Actually, this division is arbitrary, since all three types of deformation are interrelated and there are no sharp boundaries between them. This is especially true of the aftereffect. The initial stage of the aftereffect deformation begins immediately after the instantaneous elastic deformation and is usually recorded as elastic deformation. The next stage, however, is part of the total or overall creep value. Since the rate of increase of the aftereffect is greatest immediately after the load is changed, the "initial" elastic deformation of the ice which we have observed will be a function of the rate of application of load, to a considerable extent, and the time interval between the application of load and the measurement of strain. Here, time periods of even tenths of a second may exert an influence (Donchenko and Shul'man, 1949). Correspondingly, when the elastic and shear moduli are determined experimentally, their values may also depend on the rate of application of load and the duration of loading. This is also one of the reasons for the considerable discrepancies in the quantitative values of the parameters that characterize the elastic properties of ice.

THE ELASTIC MODULUS

The modulus of elasticity characterizes the resistance of ice to elastic deformation in tension or compression. If a cube of ice is subjected to unilateral compression, its relative elastic compression ϵ may be expressed by the formula:

$$\epsilon = \frac{\sigma}{E}, \tag{1}$$

where σ is the normal stress; E is the elastic modulus, which is the proportionality factor connecting the normal stress and the relative compression.

In tension, the elastic modulus is also the coefficient which connects the normal tensile stress with the relative elongation. Two methods, the static and dynamic, are used to determine the elastic modulus of ice. Essentially, the static method is the measurement of strain after the application of load, when testing ice samples in compression, tension or flexure. The dynamic method is based on the calculation of the elastic modulus on the basis of measurements of the rate of propagation of elastic vibrations in ice.

As Veinberg (1940) pointed out, the first experiments for determining the elastic modulus of ice were made at the beginning of the 19th century by Young (1820) and Bevan (1824). Later, experiments of this type were conducted by numerous other investigators. Tables 1 and 2 summarize the results of the principal experiments. As the data show, the values of the elastic modulus of ice may vary within quite broad limits. The greatest variations are observed when the static method is used. The variations are smaller when the dynamic method is used, but the average value of the elastic modulus is higher. This may be explained as follows: when the static method of investigation of ice strain is used, the deformation is not measured at the moment the load is applied but after a certain time interval. Ordinarily, this interval is small, a matter of seconds, but this is sufficient to permit a perceptible creep deformation of the ice (chiefly due to the aftereffect) together with an elastic deformation which begins immediately after the application of load. As a result, the elastic modulus determined by the measurement of total deformation does not characterize the resistance of ice to instantaneous elastic deformation, but characterizes the resistance of ice to reversible deformation after a specific time interval.

The greater the tension caused by the load, the more substantial will be the role played by the creep component of the deformation. The instantaneous elastic deformation may be regarded as directly proportional to the magnitude of the stress, but the creep rate increases considerably more intensively with an increase of stresses, approximately proportional to the square of the stress (see Chapter IV). Therefore, the magnitude of ice deformation measurable after a specific time interval following the application of load, say after 5-10 seconds, will not increase linearly with increasing stresses, but will increase more intensively. The elastic modulus calculated on the basis of strain measurements should decrease correspondingly as the stresses increase. This has been confirmed by experiments. For example, the data of V. N. Pinegin (1927) show that the elastic modulus of river ice in compression (at -3°C) decreases with increasing stresses as follows:

Stress, kg/cm^2	1.07 - 3.75	3.75-6.44	11.80-14.48	17.16-19.84
Elastic modulus, $\times 10^3$ kg/cm^3	37.5	13.7	6.0	3.4

Our experiments showed a similar picture. We determined the elastic modulus on the basis of measurements of the flexural strain of prismatic ice beams 10 x 10 x 120 cm. Figure 9 gives a schematic view of the apparatus and the loading of the beams. The experiments were designed to study the plastic properties of ice and we investigated chiefly the long-term plastic deformations. However, in passing we measured the deformations which occurred after the application of load. The first reading was made 5-10 seconds after the application of load, then after 1, 5, 10 and 30 minutes, and further after longer time intervals. Similar measurements of the deformation were made after the magnitude of the load had been changed. The elastic modulus was calculated on the basis of the first measurement of sag after the application of load or the change of load, according to formula:

$$\Delta \delta = \frac{\Delta P}{48EJ}\left[l^3 - \frac{a^2(3l-a)}{2}\right] \approx \frac{22\Delta P}{E} \qquad (2)$$

where ΔP is the change in magnitude of loading of the beam in kg, $\Delta \delta$ is the discontinuity of the sag value caused by the change in load in cm, ℓ, a and J are, correspondingly, the span of the beam, the distance between weights and the moment of inertia of the cross section of the beam.

Table 3 shows the results of one series of experiments. Three beams were tested simultaneously. Beams 1 and 2 were cut from ice of regular structure, which had formed during the calm freezing of water in an open reservoir, and beam 1 was tested such that the axes of the crystals were horizontal, perpendicular to the plane of flexure ($0 \perp f$ and $0 \perp \ell$), while beam 2 was tested such that the axes of the crystals were vertical (0ll f). Beam 3 was cut from ice of random structure frozen from a mixture of pieces of ice, snow and water. Predetermined weights were placed on the beams, the beams deformed plastically for a long period of time and then the weights were removed. After several days the last stage of loading was applied to the beams. It should be noted that the decrease of deformation upon removal of the weights corresponded approximately to the increase of deformation upon application of the loads, i. e., the values of the elastic modulus during loading and unloading were nearly identical, despite the considerable plastic deformations which occurred during the period between loading and unloading. From the data given one can see that the magnitude of the elastic modulus is determined basically by the size of the load (the stress) and decreases as the load is increased. However, a difference in ice structure and the direction of the crystal axes with respect to the action of the forces did not cause any substantial difference in the obtained values of the elastic modulus.

According to the data of V. P. Berdennikov (1948), the elastic modulus of ice is a function of the ice temperature and decreases as the temperature increases (at -40°C, $E = 95 \times 10^3$ kg/cm^2; at -2°C, $E = 90 \times 10^3$ kg/cm^2). The salinity of ice increases the temperature dependence of the elastic modulus, in this case the decrease of the elastic modulus of salty ice compared with pure ice is a function of the liquid content in the form of brine cells.

With frequent repeated loading and unloading, the elastic modulus of ice increases with the number of loadings (Pinegin, 1927; Kartashkin, 1947), and the rate of increase of the modulus decreases as the number of loadings is increased (table 4).

The elastic modulus of ice depends essentially on the density of the ice and decreases with decreasing density, e.g., Nakaya's (1958) data show the elastic modulus of ice of density 0.910-0.914 to be 90 x 10^3 kg/cm^2, while ice of density 0.900 has an elastic modulus of only (70-80) x 10^3 kg/cm^3 and ice of density 0.700 a modulus of 40 x 10^3 kg/cm^2.

Generalizing the results of the investigations, Veinberg (1940) considered the elastic modulus of ice to be (70-80) x 10^3 kg/cm^2. Later, Kartashkin (1947), on the basis of numerous experiments, established that the elastic modulus in compression, tension and flexure at temperatures from -5° to -16°C is, on an average, 40 x 10^3 kg/cm^2. At the same time, Berdennikov (1948), having determined the modulus by the acoustic method considered it to be 90 x 10^3 kg/cm^2 for monolithic ice.

Analyzing these recommended values for the elastic modulus of ice and also keeping in mind the results of experimental determinations reported above (in tables 1 and 2), we came to the following conclusions:

1. The elastic modulus of ice is, to a certain extent, indeterminable, because it is very difficult to distinguish the purely elastic deformation of ice.

Elastic deformations are those deformations of a body which disappear after the forces which caused the deformation have been removed, i.e., they are reversible deformations. The theory of elasticity contends that the deformation occurs at the moment the load is applied and disappears completely when the load is removed. However, the elastic aftereffect is strongly manifested in ice and the reversible part of the deformation does not occur immediately after the application of load, but increases over a certain period of time. Correspondingly, when the load is removed that part of the deformation does not disappear immediately. Therefore, the magnitude of the elastic (reversible) deformation of ice is a function (in contrast to the deformation of elastic bodies) of the duration of loading. Correspondingly, the elastic modulus of ice, which characterizes the relationship between the magnitude of the deformation and the load, will also be a function of time

2. If by elastic deformation of ice we mean only that part of the reversible deformation which occurs instantaneously (at the speed of sound) at the moment the load is applied, the elastic modulus should be determined only

by dynamic methods of investigation. In such a case, the most reliable value of the elastic modulus of ice will be $E \approx 90,000$ kg/cm^2. It is recommended that this value be used in calculating the elastic deformation of ice under dynamic loading.

3. When the loading effect is quite prolonged, sometimes it is expedient to take the value of the reversible deformation, which occurs during the first seconds after the load is applied and which is a more realistically perceptible value, as the initial elastic deformation. For calculations of the value of such deformation in compression, tension or flexure, the value $E \simeq 40,000$ kg/cm^2 may be taken as the elastic modulus of ice and in this case one should consider the above-mentioned relationship of this value to the stresses and other factors.

THE SHEAR MODULUS

The shear modulus characterizes the resistance of ice to shearing strain.

If an elementary cube is removed from a mass of deformable ice, its angular strain γ, in agreement with the theory of elasticity, may be expressed by the formula:

$$\gamma = \frac{\tau}{G}, \tag{3}$$

where τ is the tangential stress and G the shear modulus.

As in the case of the elastic modulus, static and dynamic methods are used to determine the shear modulus. The most frequently used static method consists in testing cylindrical or prismatic ice samples in torsion, since in this case conditions are created for pure shear.

Table 5 shows some results of experiments for determining the shear modulus of ice. The reasons for the considerable discrepancies in the values obtained are basically the same as those which arise in determining the elastic modulus.

By analogy with the recommended values of the elastic modulus, we propose the following values be accepted for the shear modulus of ice:

a) for calculating the elastic deformation during dynamic loading, $G \approx (30\text{-}34) \times 10^3$ kg/cm^2;

b) for calculating the initial deformation during prolonged loading (deformation occurring during the first few seconds of application of load), $G \approx 15 \times 10^3$ kg/cm^2.

POISSON'S RATIO

The coefficient of transverse deformation, or Poisson's ratio, is the ratio of the transverse deformation to the longitudinal deformation of a sample when compressive (or tensile) forces are applied to the sample and when the dimensions of the sample may change freely in transverse directions.

In the case of elastic deformation, Poisson's ratio is connected with the elastic modulus and the shear modulus by the following relation.

$$\mu = \frac{E}{2G} - 1. \qquad (4)$$

V. N. Pinegin (1927) has made the only direct measurements of Poisson's ratio for ice. Veinberg (1940), on the basis of an analysis of the results of these measurements and on the basis of a comparison of the propagation velocities of longitudinal and transverse vibrations in ice and also by a comparison of the elastic and shear moduli values drawn from the data of various investigations, established that the value of the Poisson ratio closest to reality is $\mu = 0.36 \pm 0.13$.

The considerable variations of the possible values of Poisson's ratio may be explained in part by the fact that the elastic deformation of ice takes place in conjunction with plastic deformation and that it is difficult to distinguish purely elastic deformation. The plastic deformation is characterized by a change in the form of the ice sample without a change of its volume, thus Poisson's ratio for pure plastic deformation is 0.5. Consequently, when the load is increased, when the plastic deformations appear more rapidly, Poisson's ratio will increase to a certain extent. Further, the anisotropy of ice also affects the value of Poisson's ratio.

B. D. Kartashkin (1947) considers that Poisson's ratio, on an average, is 0.34 for ice in the temperature range -5°C to -16°C. He bases his value on his experimental determinations of the value of the elastic and shear moduli.

B. A. Savel'ev (1953) recommends that the value 0.36 be taken for Poisson's ratio in calculations.

CHAPTER IV

THE CREEP OF ICE

Creep is the term used to define slow and steadily increasing deformation of a material under the influence of constant forces or stresses. Creep deformation in ice is irreversible (plastic) and is often regarded as slow flow.

The creep process is associated with continuous changes of form without change of volume and occurs only in presence of shear stresses, because only densification occurs under uniform hydrostatic compression. Therefore, the basic laws of creep are given first for the case of pure shear.

CREEP IN PURE SHEAR

Figure 14 shows characteristic creep curves of polycrystalline ice. In all cases, elastic deformation $\gamma_{elastic}$ takes place at the moment the shearing force is applied and creep deformation γ_{creep} begins; in the initial period the creep rate gradually decreases to a value which is a function of the value of the shear stresses.

In the case of small stresses a constant rate of creep is subsequently established, i.e., the stage of steady-state creep which may continue for an indefinite period of time (providing, of course, that the stresses, temperature and conditions of deformation are constant and lie within certain limits where the change in structure and orientation of the ice crystals may be neglected). When the stresses increase, the rate of steady creep increases and, correspondingly, possibilities arise for a more rapid change of the ice structure, which, in turn, may change the rate of deformation. Therefore, when the stresses increase, the stage of steady creep becomes limited in time and passes into the stage of accelerating creep. The greater the stress, the shorter the time interval of steady creep and the sooner accelerating creep begins. Finally, when the stresses exceed a certain limit, the distinct segment of steady-state creep disappears.

In this case, after a smooth decrease of the creep rate during the initial period to a minimum velocity, the creep rate begins to increase gradually,

becoming progressive flow, and this sets in the more rapidly the greater the stress.

One may judge the validity of the general laws of ice creep given above by the experimental creep curves shown in figure 15, obtained from long-term experiments on the torsion of cylindrical ice samples (Voitkovskii, 1957). Hollow ice cylinders 800 mm long, with an outer diameter of 120 mm and an inner diameter of 78 mm, consisting of artificially frozen polycrystalline ice were used in the experiments. In torsion the stresses were quite uniform in all the cross sections of the tubes, which permitted us to calculate the magnitude of the relative shear deformation and to establish the quantitative relationship between the magnitude and rate of shear and the value of the tangential stresses on the basis of the angular strain of the tubes. The experiments were conducted at a constant temperature and over a sufficiently long period of time. As is evident, with stresses less than 2 kg/cm^2 in all cases an approximately constant creep rate was established after 50-100 hours following the application of load. Individual experiments lasted up to 5,000 hours and even after such a long time interval there was no tendency toward an increase of the creep rate.

With a stress of 2 kg/cm^2, a constant creep rate was established after 70 hours, but after 200 hours (at a temperature of -1.8° C) the creep rate began to increase gradually. With a stress of 3 kg/cm^2 there was no clearly defined straight-line segment. During the initial 30 hours, the creep rate gradually decreased and then began to increase.

For convenience in some of the conclusions which follow, the author has used the term "the limit of prolonged creep" or τ_p to indicate the stress above which prolonged creep at a constant rate is no longer possible. In passing, it should be mentioned that this limit is somewhat arbitrary, since there is no clear stress limit to define the conditions which would permit prolonged steady-state creep, on the one hand, and transition to the stage of accelerating creep without the stage of steady-state creep, on the other. A quite prolonged stage of steady-state creep, which then became accelerating creep, was observed in a certain range of stresses. Furthermore, even in absence of the steady

creep stage, with a transition from the initial stage of decelerating creep to the stage of progressive flow (characterized in figure 14 by a bend in the curve τ_4), there was a segment where the creep rate changed comparatively little and during short-term experiments this is sometimes erroneously assumed to be steady creep.

In this connection, it has been proposed that the limit of prolonged creep should be the stress at which there is a clearly defined stage of steady creep lasting at least as long as the initial stage, namely, the stage of decelerating creep (of the order of 100 hours), i.e., when the tendency toward an increase of the creep rate may not appear sooner than 200 hours after the beginning of the deformation (with constant stresses and temperature).

According to my experimental data, the limit of prolonged creep τ_p of ice is as follows: approximately 1.6 kg/cm^2 at -1.2°C (Voitkovskii, 195?), approx. 2 kg/cm^2 at -1.8°C (see the creep curve for τ = 2 kg/cm^2 in fig. 15) and 3 kg/cm^2 at -4°C (see fig. 16).

Figure 16 shows the character of the change in the rate of steady creep as a function of the stress value. The shear-rate values taken here as the basis for the experimental works of the author on the torsion of ice cylinders with a stepwise increase of stresses are as follows: for curve 1, values taken from an earlier published work (Voitkovskii, 1957); for curve 2, the values from table 6 (ice cylinder No. 5). In figure 16a, where the experimental data are plotted against the corresponding values of tangential stresses, it is evident that the rate of shear is small when the stresses are small. When the stresses increase, the rate of shear also increases, at first smoothly and then quite abruptly. When these values are plotted on a double logarithmic scale (fig. 16b), the points lie fairly well along straight lines. This means that the relationship between the steady rate of creep $\dot{\gamma}_\infty$ and the value of the tangential stress with constant temperature may be defined by the equation

$$\dot{\gamma}_\infty = k_\theta \tau^n \qquad (5)$$

where k_θ and n are constant factors (in the given case, $n = 2-2.2$; in other experiments by Voitkovskii the n-value for ice of random structure varied from 1.6 to 2.2).

Gerrard, Perutz and Roch (1952) first proposed an equation of this type for ice on the basis of measurements of the vertical distribution of flow velocities in a glacier. They used the following values for the constants: $n = 1.5$ and $k_\theta = 10^{-8}$, where the stress is measured in bars and the rate of shear γ in seconds.

Glen (1952, 1955) used a similar equation. He established that in the case of unilateral compression of cylindrical samples of fine-grained polycrystalline ice, the ratio of the minimum strain rate $\dot{\epsilon}$ observed during the experiment and the stress value σ (within the limits 1-10 kg/cm^2) is expressed by the formula

$$\dot{\epsilon} = k\sigma^n \qquad (6)$$

where $n = 3.2-4$.

Glen's values of n are frequently cited these days in glaciological literature, but one must remember that they are too high to be representative of the prolonged steady creep of ice. In most of Glen's experiments the stresses exceeded the limit of prolonged creep and there was no stage of steady creep, while Glen compared minimum rates of deformation without consideration of the character of the creep curves.

For small stresses, the minimum rate corresponds to the rate of steady creep, but for stresses exceeding the limit of prolonged creep it merely characterizes the transition from the decelerating creep in the initial period of stress to progressive flow. Therefore, it is doubtful whether the laws of change of the minimum creep rate would remain identical in both cases. One can see this by examining the logarithmic relationship (fig. 16) between the rate of shear and the stresses, where one may see that the straight lines tend to bend upward at stresses near the limit or prolonged creep, which indicates an increase of the n-value at these stresses. This also follows from a

comparison of the experimental creep curves given in figure 15. For example, a comparison of the rate of steady creep when $\tau = 1.0$ kg/cm^2 ($\dot{\gamma} \approx 1.5 \times 10^{-5}$ 1/hr) with a minimum creep rate when $\tau = 3$ kg/cm^2 ($\dot{\gamma} \approx 50 \times 10^{-5}$ 1/hr) will show that the minimum shear rate increased more than 30-fold with a 3-fold increase of stress, i.e., approximately proportional to the cube of the stress, which corresponds to the n-value determined by Glen.

When temperature is taken into consideration, the relationship between the steady rate of creep of polycrystalline ice in pure shear and the value of the tangential stresses is defined by the equation (Voitkovskii, 1957)

$$\dot{\gamma}_\infty = \frac{K}{1 + \theta} \tau^n \tag{7}$$

where θ is the temperature of the ice (in $^\circ$C without the minus sign); K and n are factors dependent upon the ice structure (for ice of random structure, $n = 1.6$-2.2 and $K = (1.6$-$4) \times 10^5 \cdot$ degree/kg$^n \cdot$ hr.

This latter equation is obtained from the relationship between the strain rate and the temperature, if the $\dot{\gamma}_\infty$ value from formula (5) is substituted for γ_o and if $K = (1 + \theta_o) k_o$.

The increase of the deformation value during the creep process, with consideration of the initial stage may be expressed by the empirical formula (Voitkovskii, 1957)

$$\gamma_t = \gamma_{elastic} + \dot{\gamma}_\infty t + \dot{\gamma}_\infty t_o \left[1 - \frac{1}{(1 + a t)^m}\right] \tag{8}$$

where γ_t is the total deformation after any time interval t (in hours) following the application of forces (the beginning of the effect of shear stresses); $\gamma_{elastic}$ is the elastic deformation; t is the time from the moment of application of load in hours; t_o, a, m are empirical coefficients (in the described experiments they have the following values: $t_o = 30$-100 hours; $a \approx 0.5$; $m = 0.5$-1.0).

In this formula the total deformation is expressed as the sum of the elastic and creep deformations and the creep deformation is arbitrarily

divided into two components: steady creep (the second term) and transient creep (the third term). Figure 17 depicts the division of the total deformation into the indicated components. The creep rate in the initial period after loading may be expressed as the time derivative of equation (8)

$$\dot{\gamma}_t = \dot{\gamma}_{\infty} \quad 1 + \frac{a\,m\,t_0}{(1 + at)^{m+1}} \qquad (9)$$

For the case where the shear stresses exceed the limit of prolonged creep, thus far we have not been able to establish the quantitative relationship between the creep rate, the stress value and time because under such circumstances the creep rate is variable and may change within very broad limits depending on the conditions of deformation, the structure of the ice and other factors.

In a number of cases it is desirable to know the time dependence of the shear strength of the ice (resistance of the ice to shear) with a given rate of deformation. Thus far, no direct measurements have been made of the resistance with small strain rates which do not cause destruction of the ice However, one may get some idea of the nature of the change of resistance by analyzing the creep curves. The laws of the rate of change of creep of ice with time and various constant stresses, based on the experimental curves of creep (see fig. 15), are of the form represented in figure 18a. If a horizontal line corresponding to a specific creep rate (e.g., $\dot{\gamma}_4$) is drawn across this scheme, the τ-values at the points of intersection of this curve with the curves $\gamma_{t\,=\,\mathrm{const}} = f(t)$ may, under certain assumptions, be regarded as the values of the resistance at a constant rate of shear $\dot{\gamma}_4$. If these resistance values are plotted against the corresponding times (fig. 18b), one will get the curves of the change in magnitude of the resistance of ice with time and a constant rate of deformation.

As is evident from this scheme, the change of the shear strength (resistance) value is a function of the given rate of deformation. With a small rate of shear, the resistance will increase smoothly to a certain value which may be determined from equation (7), after which it will remain constant. If the shear rate corresponds to the steady-state creep rate with a stress close to the limit of prolonged creep τ_p, after an interval of constant resistance

it may decrease slightly in connection with the gradual change of ice structure. When the shear rates exceed the possible rate of prolonged steady creep, the value of the resistance after attainment of the maximum value will gradually decrease to a value close to the limit of prolonged creep.

THE INFLUENCE OF TEMPERATURE

The creep rate increases with increasing temperature. This relationship is especially strong at a temperature close to $0^{\circ}C$. On the basis of experimental data, Royen (1922) described the relationship between the value of the plastic deformation of ice (in compression) and temperature by the empirical formula

$$\epsilon = \frac{B}{1 + \theta} \qquad (10)$$

where θ is the temperature of the ice without the minus sign and B is a constant which differs in each individual case.

Experiments on the flexure of ice beams and the torsion of ice cylinders (Voitkovskii, 1956, 1957) have shown that the change in the value of the rate of steady creep with given conditions of deformation (stresses) as a function of temperatures within the range $-1^{\circ}C$ to $-40^{\circ}C$ is expressed by an empirical formula analogous to Royen's (10)

$$\dot{\gamma}_{\theta} = \frac{(1 + \theta_0)\,\dot{\gamma}_0}{1 + \theta} \qquad (11)$$

where $\dot{\gamma}_0$ is the experimentally determined rate of steady creep at any temperature θ_0; $\dot{\gamma}_{\theta}$ is the rate of steady creep at any temperature θ.

This formula is acceptable for cases of pure shear as well as for other types of deformation (compression, tension, flexure, deformation with a complex state of stress), provided there is a stage of steady-state creep and that the stress values remain the same at all points with a change of temperature, i.e., provided a change of temperature does not cause a re-distribution of the internal stresses.

Figure 19 shows the results of one of the experiments on the influence of temperature. The experiment consisted in the following: Weights

were placed on a 10 x 10 x 120 cm beam of ice of random structure (the diagram of the apparatus and the loading of the beams is shown in fig. 9), after which the sag values were measured systematically, which made it possible to determine the rate of sag of the beam. The temperature in the room where the experiment was conducted was kept approximately constant during the time required to establish a constant sag rate, after which it was changed and the steady rate of sag was again determined. The temperature was varied from $-1^{\circ}C$ to $-40^{\circ}C$. In figure 19, the experimentally determined rates of sag of the beam at the corresponding temperatures are indicated by the circles. As can be seen, within this temperature range these circles correspond satisfactorily to the curve which depicts graphically the relationship between the creep rate and the temperature, in agreement with formula (11), if we assume that $\gamma_o = 4 \times 10^{-3}$ cm/hr at $-3.5^{\circ}C$ ($\theta = 3.5$).

The examined relationship between the rate of steady creep and the temperature may be employed to determine the influence of temperature on plastic deformation after a specified time interval during the initial period after application of load, as follows from an analysis of formula (8). In this case it should be kept in mind that this can be accepted fully only when the stresses do not exceed the limit of prolonged creep. If the stresses are large, however, the temperature relationship may be more complex, since the very limit of prolonged creep is a function of temperature; furthermore, the creed rate, as indicated above, is very variable under these conditions and depends on many factors.

Some investigators have proposed other equations for the relationship between the creep rate of ice and temperature, but these equations have not been properly verified by experimental data. For example, Glen (1955) accepted the equation

$$\dot{\epsilon} = Ae^{-\frac{Q}{RT}} \qquad (12)$$

as an expression of the relationship between the compressive strain rate $\dot{\epsilon}$ and the absolute ice temperature T. Here A is a constant, R the gas constant and Q the activation energy (a heat of activation).

Earlier, A. R. Shul'man (1948) declared it possible to use such an equation to express the relationship between the viscosity of ice and temperature. Later, Jellinek and Brill (1956) used a similar equation. However, Voitkovskii's experimental data indicate that the above equation (12) does not reflect the actual abrupt increase of creep velocity with temperature above -5°C to -3°C; therefore, we do not recommend its use.

CREEP UNDER THE SIMULTANEOUS INFLUENCE OF NORMAL AND TANGENTIAL STRESSES (IN A COMPLEX STRESSED STATE)

Uniform hydrostatic pressure does not exert a substantial influence on the character and the rate of creep. Rigsby (1958), in conducting experiments on the shearing of ice crystals at pressures up to 306 atm, established that the rate of shear deformation is practically independent of pressure if the difference between the ice temperature and the melting point (which varies as a function of pressure) remains constant. If the temperature of the deforming ice remains constant, the shear strain rate will increase somewhat with increasing pressure, but this increase becomes substantial only at quite considerable pressures. For example, at a pressure of 306 atm, the shear rate approximately doubled. Thus, for the most part hydrostatic pressure is expressed merely as follows: it reduces the melting point of ice and its influence on the creep rate is equal to the effect of an actual increase of ice temperature during the deformation of the ice

Irregular or unilateral pressure, as distinct from hydrostatic, has a substantial influence on the creep rate. To arrive at a quantitative estimate of its influence, I conducted long-term experiments on the simultaneous torsion and longitudinal compression of hollow ice cylinders consisting of ice having randomly oriented crystals. During the experiment, normal and tangential stresses, distributed quite uniformly and identical in magnitude, were created in all cross sections of the ice cylinder. This made it easy to determine the relative angular and longitudinal strains. The values of the torque and longitudinal force were chosen such that the influence of normal stresses on creep could be traced for given constant values of the shear stresses.

In all, six ice cylinders were tested. Various combinations of the simultaneous effect of normal and tangential stresses were created for each

cylinder (normal from 0-5 kg/cm^2, tangential from 0.75-2.5 kg/cm^2). Each stage of loading was continued for at least 200 hours, so that the rate of steady creep could be determined.

Figure 20 shows the order of magnitude of the change in the value of the stresses and the character of the creep curve for one of the experiments. The results of the remaining experiments are given in table 6. As can be seen from figure 20, a constant rate of creep was established for all combinations of stresses except the case where the limit of prolonged creep was exceeded. In this latter case, the rate of shear did not depend solely on the value of the tangential stresses but also on the value of the normal stresses acting in the shear plane and increased as the normal stresses increased. Similarly, the rate of longitudinal compression at a constant compressive stress increased as the tangential stresses increased in the planes perpendicular to the direction of compression. Thus, the steady rate of creep in the complex stressed state proved to be a function of both the tangential and the normal stresses.

Analysis of the experimental data shows that the steady-state creep of polycrystalline ice in the complex stressed state may be expressed with the aid of equations of the theory of plastic flow (Sokolovskii, 1950), assuming that the value of the shear strain rate of ice L is a specific function of the value of the tangential stresses S:

$$L = f(S) \tag{13}$$

where $L = \sqrt{\frac{2}{3}\left[(\dot{\epsilon}_x-\dot{\epsilon}_y)^2 + (\dot{\epsilon}_y-\dot{\epsilon}_z)^2 + (\dot{\epsilon}_z-\dot{\epsilon}_x)^2\right] + \dot{\gamma}^2_{xy}+\dot{\gamma}^2_{yz}+\dot{\gamma}_{zx}}$;

$$S = \sqrt{\frac{1}{6}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2(\sigma_z-\sigma_x)^2\right] + \tau^2_{xy} + \tau^2_{yz} + \tau^2_{zx}}.$$

Here it is assumed that the stressed state at any point of the body is characterized by the stress components σ_x, σ_y, σ_z, σ_{xy}, σ_{yz} and σ_{zx}, while the rate of deformation is determined by the six components of the strain rates in these same axes $\dot{\epsilon}_x$, $\dot{\epsilon}_y$, $\dot{\epsilon}_z$, $\dot{\epsilon}_{xy}$, $\dot{\epsilon}_{yz}$, $\dot{\epsilon}_{zx}$.

For the case of pure shear, when $L = \dot{\gamma}$ and $S = \tau$, this relationship, in agreement with eq. (7), should appear as follows

$$L = \frac{K}{1+\theta} S^n \tag{14}$$

Proceeding from the generally accepted hypothesis of proportionality of the principal shear rates and the principal tangential stresses, the ratio of the deviator of the strain rates $D_{\dot{\epsilon}}$ to the deviator of the stresses D_σ should be proportional to the ratio of the value of the shear strain rates to the value of the tangential stresses:

$$\frac{D_{\dot{\epsilon}}}{D_\sigma} = \frac{L}{2S} \tag{15}$$

where $D_{\dot{\epsilon}} = \begin{vmatrix} \dot{\epsilon}_x & \frac{1}{2}\dot{\gamma}_{xy} & \frac{1}{2}\dot{\gamma}_{xz} \\ \frac{1}{2}\dot{\gamma}_{yx} & \dot{\epsilon}_y & \frac{1}{2}\dot{\gamma}_{yz} \\ \frac{1}{2}\dot{\gamma}_{zx} & \frac{1}{2}\dot{\gamma}_{zy} & \dot{\epsilon}_z \end{vmatrix}$;

$$D_\sigma = \begin{vmatrix} \sigma_x - \sigma_{av} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_{av} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_{av} \end{vmatrix}.$$

Substituting the L-value from (14) in this equation, we get the generalized relationship between the rate of steady creep of ice in the complex stress state and the temperature and stresses in the form:

$$D_{\dot{\epsilon}} = \frac{KS^{n-1}}{1+\theta} \frac{D_\sigma}{2}; \tag{16}$$

$$\dot{\epsilon}_{x(y,z)} = \frac{KS^{n-i}}{1+\theta} \frac{\sigma_{x(y,z)} - \sigma_{av}}{2};$$

$$\dot{\gamma}_{xy(yz,\ zx)} = \frac{KS^{n-1}}{1+\theta} \tau_{xy\ (yz,\ zx)}.$$

Table 6 gives the actual values of the relative increase of the shear rate due to normal stresses and also shows the values calculated on the basis of formulas (16). A comparison of these figures shows the agreement of the given experimental proposals concerning the generalized relationship between the creep rate and the stresses and temperature.

Figure 21 shows graphically the results of determining the steady rate of shear $\dot{\gamma}_\infty$ for cylinder 6, plotted on a double logarithmic grid against corresponding values of the rate of change of tangential stresses. The dashed line here characterizes the relationship between the rate of shear and the value of the tangential stresses in pure shear, while the tangent of the slope angle of this line characterizes the magnitude of the coefficient n in formula (7). The points corresponding to shear rates with identical values of tangential stress but with different values of normal stress are connected by solid lines. The points lie along straight lines, the tangents of the slope angle of these lines are approximately a unit smaller than the tangent of the slope angle of the dashed line. This means that the rate of shear and the rate of change of stresses with constant tangential stresses in the shear plane are related by a power function of the type $\dot{\gamma}_\infty = aS^{n-1}$, corresponding to formula (16).

Employing relationship (16), one may calculate the rate of steady creep of ice with different types of deformation: tension, flexure, compression, and the more complex types of deformation. Then, by comparing the actually observed rates of creep with the corresponding calculation formulas, one may determine K and n, which characterize ice creep. Thus, on the basis of analytical calculations of the sag rate of ice beams, proceeding from the relationship (16) and the experimental data on the bending of beams, I found the following values for ice of random structure: $n = 1.8$ and $K = 2.3$-2.5 cm^{2n}. degree/kg^{n}. hr (Voitkovskii, 1957), which fully corresponds to the values of these coefficients in pure shear. This is another indication of the applicability of eqs. (16).

As experiments have shown, the basic features of the laws of the creep of polycrystalline ice in a complex stressed state are the same as those described above for pure shear (see fig. 14), except that the limit of prolonged creep in this case corresponds to a value of tangential stresses which, with some approximation, may be regarded as the maximum shear stress. One may

assume that the calculated coefficients K and n, characterizing the creep rate and the limit of prolonged creep, are the same as in the case of pure shear*.

Here we should emphasize, as we did in describing creep in pure shear, that the proposed generalized relationship (16) is completely valid only for cases where the stage of steady-state creep is observed. Quantitative laws have not yet been established for the change of the rate of deformation during the stage of accelerating creep.

The change in the magnitude of ice deformation during the initial period of creep after the application of load may be calculated by an empirical formula similar to formula (8), substituting in it the steady-state creep in a given stress state calculated on the basis of (16) in place of $\dot{\gamma}_{\infty}$. For example, for the case of unilateral compression (or tension), a change in the value of the relative compressive (or tensile) strain is expressed by the formula

$$\dot{\epsilon}_{\infty} = \frac{K}{1+\theta} \frac{\sigma^{n}}{3^{\frac{n+1}{2}}} ; \tag{17}$$

$$\epsilon \approx \frac{\sigma}{E} + \frac{K}{1+\theta} \frac{\sigma^{2}}{5.2} t \left(1 + \frac{at_o}{1+at}\right),$$

(taking n = 2 and m = 1).

Sometimes Royen's (1922) empirical formula

$$\epsilon = \frac{c\sigma \sqrt[3]{t}}{1+\theta} \tag{18}$$

is still used to calculate the value of ice deformation in compression and the thermal stresses in ice. In Royen's formula ϵ is the relative compressive

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* I assume that the normal stresses and especially hydrostatic pressure may raise the limit of prolonged creep of ice somewhat, however, this has not yet been confirmed experimentally.

strain and $c \cong (6\text{-}9) \times 10^{-4}$. However, Royen's formula has a number of substantial defects. First, it assumes a linear relationship between strain and stresses, while the strain actually increases approximately in proportion to the square of the stress. Secondly, according to the formula the strain rate should decrease continuously, while actually it decreases only during the initial period and then a constant creep rate is established or accelerating creep begins. Therefore, we do not recommend its use in calculations.

VISCOSITY

Most researchers have examined the creep of ice as viscous flow, with a velocity that can be characterized by the viscosity coefficient. Therefore, the study of the creep of ice usually amounts to a determination of the viscosity coefficient.

The coefficient of viscosity characterizes the internal friction which appears during the relative movement of the adjacent layers of a body and which depends on the forces of adhesion between the molecules. It may be examined as the resistance of the body at a given moment of its deformation per unit surface of the shear layer and per unit angular velocity of the shear (Veinberg, 1940). The coefficient of viscosity is measured in poises:

$$1 \text{ poise} = \frac{1 \text{ dyne. sec}}{\text{cm}^2} = \frac{0.00102 \text{ g. wt.} \cdot \text{sec}}{\text{cm}^2}$$

Results of determinations of the viscosity coefficient can be found in the works of B. P. Veinberg (1906), Deeley (1908), Lagally (1930), P. P. Kobeko (1946), V. V. Lavrov (1948), A. R. Shul'man (1948), S. K. Khanina (1949) and others. The data obtained are so contradictory (individual values of the coefficient of viscosity for ice vary from 10^{10} to 10^{15} poise, i.e., they may differ by a factor of 100,000), that they are not conducive to establishing a definite law of change of the coefficient of viscosity. The discrepancies have been ascribed chiefly to the influence of the ice structure and the direction of the deformation with respect to the optic axes of the ice crystals.

Actually, however, the main reason for the large discrepancies between the determined values of the viscosity coefficient is that this coefficient,

when applied to ice, is not a specific physical constant but is arbitrary, since it may vary within wide limits depending on the magnitude and duration of the stresses, in addition to their dependence upon the structure and orientation of the crystals and the temperature. Furthermore, crude methodological errors are commonly made since the above factors are usually ignored in determining the coefficient of viscosity.

The coefficient of viscosity of ice is usually calculated on the basis of measurements of the rate of deformation (strain), proceeding from the assumption that ice satisfies Newton's law of viscosity, i.e., that the relationship between the stress values and the rate of strain is linear. The works of Glen (1952, 1955) were the first to show that ice does not satisfy Newton's law of viscosity.

The works of a number of investigators, namely, Glen (1952, 1955), Gerrard, Perutz and Roch (1952), Haefeli (1952), Steinemann (1954), Voitkovskii (1956, 1957) and others have proved decisively that the relationship between the stress values and the strain rate is not linear and that the coefficient of viscosity of ice is not a specific physical constant but a variable dependent upon many factors.

The relationship between the strain rates and the stresses in ideally viscous flow (n = 1) can be expressed in general form by the equation

$$D_{\epsilon} = \frac{D_{\sigma}}{2\eta}, \tag{19}$$

where η is the coefficient of viscosity.

If the stage of steady creep is examined arbitrarily as viscous flow, the coefficient of viscosity of ice for this case, using formulas (16) and (19), should be

$$\eta = \frac{1 + 0}{KS^{n-1}}, \tag{20}$$

i.e., it depends on three factors: ice structure, characterized by the coefficients K and n, temperature, and the intensity (rate of change) of the tangential stresses.

In cases where there is no prolonged stage of steady creep, the coefficient of viscosity becomes a completely indeterminable value. Therefore, the coefficient of viscosity of ice may be examined only as an arbitrary value characterizing the relationship between the stress values and the creep rate with given strain conditions at a given moment of time.

CREEP WITH VARIABLE LOADING

When the load is changed, the rate of deformation changes abruptly and within a certain time after the load is changed (up to 100 hours) the character of the change of the deformation and the rate of change are functions of both the magnitude of the acting stresses and the magnitude of the stresses before the load was changed, due to the appearance of an elastic aftereffect (see Chapter II).

If the load on the ice at a specific moment of time t_1 is changed and the stresses consequently change from τ_1 to τ_2, the further behavior of the deformation may be described by the formula (Voitkovskii, 1956)

$$\gamma_t = \gamma(\tau_1, t) + \gamma(\tau_2, t-t_1) - \gamma(\tau_1, t-t_1), \quad (21)$$

where γ_t is the magnitude of the deformation (strain) at any time, γ_{τ_1}, t is the deformation which would have occurred at time t if the stresses had not changed (with τ_1); $\gamma(\tau_2, t-t_1)$ is the deformation calculated on the basis of formula (8) for $t-t_1$ with stress τ_2; $\gamma(\tau_1, t-t_1)$ is the same as above with stress τ_1.

Figure 22 shows a scheme for calculating the magnitude of the deformation on the basis of this formula. Here it is assumed that the total deformation at any moment in time after the change of stress may be expressed as the sum of two arbitrary values. The first of these is the strain which would occur at the time of interest to us if the stresses did not change. The second is the difference between the calculated strain values for the new and the old stresses. In the diagram, which shows a case of the reduction of stresses, this difference is negative and is shown by the shaded portions.

The subsequent changes of stress may be calculated analogously. In the case of unilateral compression of the ice, with a change of stress from σ_1 to σ_2 (stress σ_1 acted during time interval t_1), the further change of the relative compressive strain, in agreement with (17) and (21), is expressed as:

$$\epsilon \approx \frac{\sigma_2}{E} + \frac{K}{5.2\,(1+\theta)} \left\{ \sigma_1^2\, t \left(1 + \frac{at_{01}}{1+at}\right) + (\sigma_2^2 - \sigma_1^2)(t - t_1) \left[1 + \frac{at_{02}}{1 + a\,(t - t_1)} \right] \right\}. \tag{22}$$

It should be noted that the values of the empirical coefficient t_o may vary within quite broad limits. In cases of initial loading and an increase of stresses t_o = 30-100 hours, as shown by my experiments, but when stresses are reduced t_o = 5-10 hours, and with repeated increases of stresses t_o = 5-30 hours (Voitkovskii, 1956). Therefore, in formula (22) two values of the coefficient t_o - t_{01} and t_{02} are introduced, which should be assigned according to the specific conditions of deformation.

If the sign of the stresses changes when the direction of the loading is reversed, the creep rate usually increases. For example, in my experiments (Voitkovskii, 1957), the sag rate of the ice beams approximately doubled with the same loading values when the direction of sag was changed. This can be explained as follows. Usually, the creep of ice is associated with some breaking of bonds and in a number of cases with the partial destruction of the crystals in the shear surfaces, therefore the shear strength of the ice decreases in the direction opposite the initial direction.

RELAXATION

The term relaxation is used to define the decrease of the resistance of a body during its steady deformation. The laws of relaxation of stresses in ice have not yet been studied sufficiently. B. P. Veinberg (1907) held that the relaxation of stresses in ice obeys Shvedov's law, which states that the stresses decrease exponentially when the strain is constant

$$\sigma - \sigma_{\lambda} = (\sigma_{o} - \sigma_{\lambda})\, e^{-\frac{t}{\alpha}} , \qquad (23)$$

where σ_o is the stress at the initial moment; σ_λ is the elastic limit and α is the relaxation time.

However, this law is not very applicable to ice, because the assumptions accepted initially in deriving the formula are not applicable for the deformation of ice. The reason for this is that the elastic limit of ice is close to 0, while the relaxation time, which is related to the coefficient of viscosity, is variable.

Kartashkin (1947), in analyzing the results of his experiments, came to the conclusion that the relaxation of stresses can be expressed roughly by a formula similar to formula (23), but with the final value of the stress after a specified period of relaxation substituted for the elastic limit. However, this modification of the formula leaves quite a bit to be desired since usually the final stress value is not known. Furthermore, the relaxation time α remains an indefinite value.

We have used the above-described laws of ice creep and the general postulations about relaxation to derive a more acceptable relaxation formula. The postulations about relaxation can be summarized as follows: the total deformation value at any moment may be examined as the sum of the elastic and plastic deformations, further, during the process of relaxation the reduction of stresses is due to the gradual reduction of the elastic deformation and the addition of the plastic deformation to this same value, according to the system:

$$\gamma_{initial} = \gamma_{elastic\ (t)} + \gamma_{plastic\ (t)} = const \quad (24)$$

or

$$\dot{\gamma}_{elastic} + \dot{\gamma}_{plastic} = 0,$$

where $\gamma_{elastic}$ and $\gamma_{plastic}$ are, correspondingly, the elastic and plastic deformations at any moment in time t and $\dot{\gamma}_{elastic}$ and $\dot{\gamma}_{plastic}$ are the rates of elastic and plastic deformation.

The rate of plastic deformation of ice (creep) is determined by the intensity of the tangential stresses, thus the condition of relaxation, in agreement with the above scheme, may be expressed as a sum of the rates of change of elastic and plastic shear strain amounting to 0

$$L_{elastic} + L_{plastic} = 0 \quad (25)$$

The rate of change of elastic shear strain, according to the theory of elasticity, is

$$L_{elastic} = \frac{1}{G}\frac{dS}{dt} \quad (26)$$

where G is the shear modulus.

Let us assume that the rate of plastic deformation during the relaxation process is equal to the creep rate according to (9). Substituting the value of the shear strain rate L for the value of the rate of angular strain $\dot{\gamma}$ in formula (9), we get [with consideration of (14) and m = 1] the following values for the rates of change of plastic deformation:

$$L_{plastic} = \frac{K}{1+\theta} S_{plastic} \left[1 + \frac{at_o}{(1 + at)^2}\right] \quad (27)$$

Substituting the values of $L_{elastic}$ and $L_{plastic}$ in eq. (25), dividing the variables and integrating, we get

$$\frac{1}{G}\frac{dS}{dt} + \frac{K}{1+\theta} S_{plastic}\left[1 + \frac{at_o}{(1+at)^2}\right] = 0;$$

$$\int_{S_{initial}}^{S_t} \frac{dS}{S_{plastic}} + \frac{KG}{1+\theta}\int_0^t \left[1+\frac{at_o}{(1+at)^2}\right] dt=0; \qquad (28)$$

$$S_t = \frac{S_{initial}}{\sqrt[n-1]{1+\frac{KG}{1+\theta}(n-1)\, S_{initial}^{\,n-1} t \left(1+\frac{at_o}{1+at}\right)}}.$$

The last equation is also the equation of relaxation of stresses. In agreement with the experimental data on the creep rate of ice with variable loading, the value of the empirical coefficient t_o should be from 5 to 30 hours. If the relaxation begins immediately after the initial elastic deformation, the t_o-value will be of the order of 30 hrs. However, if the ice experiences stresses and creep deformation before relaxation begins, the t_o value will decrease to 5-10 hours.

For the case of uniaxial compression of ice, assuming $G = 15,000$ kg/cm^2; $n = 2$; $K = 3 \times 10^{-5}$; $a = 0.5$; $t_o = 10$; the relaxation equation will assume the form

$$\sigma_t = \frac{\sigma_{initial}}{1+\frac{0.26}{1+\theta}\sigma_{initial} t \left(1+\frac{5}{1+0.5t}\right)} \qquad (29)$$

Figure 23 shows the curves of relaxation of stresses calculated by this formula for the initial stresses of 10, 5 and 3 kg/cm^2 at -1.6°C. As one can see from the graph, the character of the relaxation curves corresponds to the experimental relaxation curves obtained by Kartashkin (see fig. 13). However, the relaxation formula which we have proposed requires further experimental verification and refinement.

THE DEFORMATION OF ICE IN AN INHOMOGENEOUS STRESSED STATE

A homogeneous stressed state in ice is created only in experiments with unilateral compression or elongation of samples of regular form or in experiments with torsion of samples made of thin-walled tubes. Experiments of this type are required to establish the laws of ice creep and to evaluate these laws quantitatively. In all other cases, after a load has been applied to a mass of ice or to an ice sample, an inhomogeneous stress is created in which the magnitude and direction of the main stresses differ at different points.

The appearance of an inhomogeneous stressed state during the deformation of ice causes a redistribution of internal stresses, since the laws of the distribution of stresses differ for initial elastic deformation and for creep. In the stage of elastic deformation there is a linear relationship between the magnitude of the deformation and the stress, while in the case of creep the magnitude (rate) of deformation increases considerably more rapidly than the stress. Rapid plastic deformation can occur in places where higher shear stresses develop during initial elastic deformation, but since the deformations are continuous this will be prevented to a certain extent by the more slowly deforming (less stressed) adjacent sectors. To summarize, the stresses in the more stressed portions will decrease in part (relax) due to the increase of stresses in the nearest less stressed portions.

An example of the redistribution of stresses for the case of the bending of an ice beam (Voitkovskii, 1957) is illustrated in figure 24. The dashed line here shows the distribution of stresses in the cross section of a beam at the initial moment after application of load (during elastic deformation). The solid line shows the distribution of these same stresses during steady-state creep. The maximum stresses at the upper and lower surface of the beam decreased, while the stresses in the middle part increased.

The rate of redistribution of internal stresses depends on the magnitude and the inhomogeneity of the initial shear stresses in the deforming mass of ice. The more intense the shear stresses are, the more rapid will be the plastic deformation and the more rapid can be the relaxation of stresses. An especially intensive redistribution of stresses occurs in cases where shear

stresses exceeding the limit of prolonged creep appear at the initial moment after application of load, since conditions are created for progressive flow at points of increased stress. Shear stresses exceeding the limit of prolonged creep may be preserved only during a limited time period. Under such stresses, the deformable volume may disintegrate as a result of accelerating creep or the large stresses decrease due to their redistribution.

The redistribution of internal stresses causes a reduction of the rate of plastic deformation of the loaded sample or of the mass of ice as a whole. Therefore, in the case of an inhomogeneous stressed state, the initial, gradually-decreasing transient stage of creep may be very prolonged. This may be seen in the case of the insertion of rigid dies into ice. For example, according to the experiments carried out by Votiakov in 1958 in the Laboratory of Soil Mechanics of the Northeastern Section of the Institute of Permafrost, Academy of Sciences of the USSR*, the rate of insertion of a die into ice gradually decreased over approximately 1,000 hours and only then was a constant rate of insertion established (fig. 25). Here, the period of time before the establishment of a constant rate of deformation was 10-20 times longer than in the case of pure shear.

In examining the problem of the redistribution of internal stresses in a deformable mass of ice, one must always strictly distinguish between the shear stresses and the stresses of hydrostatic compression. The redistribution of internal stresses is due to processes of creep and relaxation, which may exist only in presence of shear stresses and in practice is independent of the stresses of hydrostatic compression. Therefore, the presence of large and irregularly distributed normal stresses do not in themselves indicate the possibility of a redistribution of stresses. For example, there can be no redistribution of internal stresses under the middle part of a large heavily loaded die, despite the great stresses, since such ice will be compressed hydrostatically. The main region of redistribution of stresses will be the areas around the edges of the die where greater shear stresses appear.

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THE INFLUENCE OF ICE STRUCTURE ON ICE CREEP

Ice crystals have sharply defined mechanical anisotropy. Therefore, all the above-indicated quantitative relationships and creep characteristics are fully applicable only to the deformation of polycrystalline ice with randomly oriented crystals in volumes larger than the dimensions of the individual crystals, when ice may be regarded as a solid isotropic body. For ice with a clearly defined crystal orientation, the basic creep laws remain unchanged, however, the creep rate may vary as a function of the direction of the acting shear stresses with respect to the direction of the crystal axes. As a result, the coefficients n and K, which characterize steady-state creep [formulas (7) and (16)], may differ from those previously indicated, mostly they will be larger. This may be explained as follows: for oriented structure, in individual cases there is an increased possibility of the deformation of ice without disintegration of its individual crystals, for example, when the shear planes coincide with the basal planes of the crystals or with the contacts between crystals. In such cases, the internal resistance to deformation may be smaller than in the case of the deformation of ice of random structure and correspondingly the creep rate may be greater. In our experiments on the torsion of ice cylinders and the bending of ice beams with specific orientation of the crystals (Voitkovskii, 1957), the n-value in individual cases reached 2.4 and the K-value 9×10^{-5}.

The influence of ice structure is expressed particularly strongly in the case of shear stresses which exceed the limit of prolonged creep, when there is accelerating creep.

CHAPTER V

THE ULTIMATE STRENGTH OF ICE

The ultimate strength or the breaking point of any material is the stress at which the material ruptures. For ice this magnitude is conditional to a certain degree, since the rupture of ice is due not only to the attainment of a certain critical stress, but, in view of the considerable role played by creep phenomena, the beginning of ice rupture and the magnitude of internal stresses corresponding to this moment depend substantially on the rate of application of load, the conditions of deformation and other factors. This is also one of the reasons for the large fluctuations of the ultimate strength values of ice determined by various investigators.

The ultimate strength of ice usually is defined as the greatest stress (resistance) in the test sample of ice before its rupture due to "rapid" loading. The ultimate strength of ice depends on the type of deformation, thus, there are ultimate strengths in compression, tension, flexure and shear. Due to the specific nature of the mechanical properties of ice, the determinable values of its ultimate strength are somewhat different in nature and design than the ordinarily applied ultimate strengths of various materials. The ultimate strength characterizes the ultimate resistance of the material to external forces and ordinarily is used to determine the possible loads which can be supported by some structure or structural unit made of the given material. However, when ice is used as a construction material or as a bearing foundation, the permissible load is determined from the permissible magnitude and rate of plastic deformation of ice under the specific conditions and not by the magnitude of the permissible stresses, which should be less than the ultimate strength (Voitkovskii, 1954).

Data on the magnitude of the ultimate strength of ice are required basically in cases where the problem of combating ice is examined. For example, such data are required for calculating structures that are subject to ice action and for determining the possible forces of interaction between the

ice and the structure. The magnitude of ultimate strength characterizes the force required for the mechanical destruction (disintegration) of the ice.

The crushing strength of ice. Table 7 gives the basic results of experiments to determine the ultimate strength of ice under unilateral compression conducted by various investigators. From these data it is evident that the resistance of the ice varies within broad limits depending on the structure of the ice, the orientation of the crystals with respect to the direction of compression, the temperature and other factors. One also observes a considerable scatter of ultimate strength values, even in tests of samples of analogous structure under identical conditions.

It should be noted that the magnitude of the ultimate strength of ice depends to a considerable extent on the conditions of deformation, namely, the dimensions of the test samples and the rate of application of load (or the rate of deformation), which have received little attention from most investigators. According to the data of N. A. Tsytovich (see Tsytovich and Sumgin, 1937) the ultimate strength in compression of identical ice samples varied within the following limits as a function of the rate of increase of load:

Rate of increase of load, $kg/cm^2 \cdot min$	20	36	50
Ultimate strength in compression, kg/cm^2	60	37	24

K. N. Korzhavin (table 8) noted similar phenomena. He established that an increase of the rate of deformation leads to a reduction of the ultimate strength, in which case the influence of the rate of deformation is particularly strong at low temperatures, and the influence decreases as the temperature approaches 0°C. Korzhavin (1951) represents the relationship between the ultimate strength and the rate of relative deformation S (within the limits 0.0007-0.0417 sec^{-1}) by the empirical formula

$$\sigma = \frac{a}{\sqrt[3]{S}} \qquad (30)$$

where a is an empirical coefficient (at -3°C, a = 3.1; at 0°C, a = 2.5).

Let us note that this type of relationship may exist only with relatively large rates of deformation, since the reverse picture is observed in the case of small deformation rates: an increase in the deformation rate leads to an increase in the resistance (see fig. 18). In compression, an ice sample often begins to disintegrate before the stresses in it reach the breaking point. For example, in compression tests of samples of underground ice (L. S. Khomichevskaia, 1940) it was noted that cracks began to appear in the samples at stresses 2-3 times less than the breaking point (ultimate strength). An ice sample in which cracks have formed may disintegrate in time without an increase of stresses, i. e., under the more or less prolonged effect of the same stress under which the cracks formed, with the following result. The ultimate strength of ice may decrease substantially under the prolonged effect of loading or a very low rate of increase of load.

To date not enough study has been devoted to the influence of the size of the test samples on the magnitude of the ultimate strength. Comparing the results of tests of 10 cm and 20 cm cubes of ice, K. N. Korzhavin (1940) noted an increase in the ultimate strength of the large ice samples. However, this increase is not always observed. For example, Butiagin (1955) asserts that in experiments carried out under natural conditions of destruction of an ice cover, the ultimate strength of various types of deformation of large ice samples was less than that of the small samples tested.

The ultimate strength of ice is a function of temperature and increases as the temperature decreases. This relationship may be expressed by the empirical formula (Korzhavin, 1940):

$$\sigma = A + B\theta, \qquad (31)$$

where θ is the negative temperature of ice in $^{\circ}$C (without the minus sign); A and B are empirical coefficients (for the case of the crushing of 10 cm cubes of ice at a rate of $v = 2$ cm/min in the temperature interval 0°C to -10°C, $A \approx 15$ and $B \approx 3.4$).

The ultimate strength of ice in compression in the direction of the crystal axes $\sigma_{\parallel}$ is usually greater than it is in the direction perpendicular to the crystal axes $\sigma_{\perp}$. For example, according to Korzhavin's data (1951)

the ratio $\sigma_{\parallel} / \sigma_{\perp}$ for the ice cover without conspicuous indications of the weakening of the bonds between the crystals at a temperature of $0^{\circ}C$ to $-3^{\circ}C$ is, on an average, 1.3-1.5. During the spring thaw, when there is a perceptible weakening of the bonds between the crystals, this ratio increases and may reach 3.6.

The strength of a natural ice cover is not uniform vertically. The ice is strongest in the central part of the cover and weakest in the lower part.

The tensile strength of ice. This depends basically on the same factors as the crushing strength, except that the ultimate tensile strength is considerably smaller than the crushing strength and varies within smaller limits (table 9). Furthermore, various inclusions and structural irregularities, which may become centers of destruction, have a great influence on the tensile strength value. In compression an ice sample may permit a further increase of load after cracks have appeared, but in tension the ice sample usually breaks without preliminary crack formation.

The fracture strength of ice is determined by bending ice samples or a portion of the ice cover, for example, by bending strips of ice cut from the ice cover, the so-called "ice keys" (literally, "piano keys of ice," tr.). The most probable centers of rupture are the breaks in the tension zone or the shearing (cleaving) at points of greatest tangential stresses and, correspondingly, the beginning of rupture should be determined by the attainment of critical tensile stresses or critical shear stresses.

The ultimate flexural strength of ice is usually defined as the maximum tensile stress in the bending sample of ice before its destruction, calculated on the basis of formulas for a linearly elastic body. In such an approach, the determinable magnitude of the maximum stress is greater than the actual stress in the test sample, since during flexural testing of ice plastic deformations appear along with elastic deformations and in the case of these plastic deformations, the distribution of stresses in the flexure sample changes compared with the distribution of stresses in linear elastic deformation. This change tends toward a reduction of the maximum stresses (see fig 24).

Thus, the determinable ultimate strength is an arbitrary value to a certain extent, somewhat greater than the actual maximum stress in fracture In our opinion, this can be explained by the following. According to the data

of most investigators, the ultimate flexural strength of ice (table 10) is greater than the ultimate tensile strength, since it is not very likely that the actual tensile stresses in a flexure sample would increase beyond the ultimate tensile strength

The concepts outlined above do not prevent the practical use of the given ultimate strength values for computing the conditions of ice fracture, considering that the over-valuation of the str[illegible]ses allowed in determining the ultimate strength will be compensated by a corresponding under-valuation of the actual stresses compared with the calculated stresses in the cases of interest to us. One need but observe the following. In the calculations one must proceed not from the value of the actual stresses, but from the calculated stresses, and regard ice arbitrarily as a linearly elastic body.

The magnitude of the flexural strength of ice depends very substantially on the size of the bending samples and the rate of application of load. For example, according to the data of I. P. Butiagin (1955), the ultimate strength of small samples (7 x 7 cm and 10 x 10 cm in cross section and 50 cm long) on an average is three times greater than the ultimate strength of large strip samples cut from the ice cover. V. V. Lavrov (1958) attempted to give a theoretical explanation of the influence of the size of the ice test samples on the magnitude of the ultimate strength and to present corresponding calculation formulas. Lavrov's theoretical premises are somewhat debatable, so we have not given his formulas here.

For example, according to Lavrov's formulas, the ultimate flexural strength of a sample 4.5 cm thick and 35 cm long would be 23 kg/cm^2, while it would decrease to 9 kg/cm^2 for a sample 34 cm thick and 250 cm long. Similarly, while the ultimate fracture strength of an ice cover 0.35 cm thick would be 21 kg/cm^2, it would be only about 7 kg/cm^2 for an ice cover 1-2 m thick.

K. N. Korzhavin treated the relationship between the fracture strength and the rate of loading. According to his data (see table 10), an increase of the bending rate from 2 to 20 cm/min decreased the ultimate strength from 9.2 to 3.6 kg/cm^2.

Orlov (1940) noted that the fracture strength of an ice sample in water is somewhat less than the fracture strength of "dry" ice. The bending of ice "keys" has shown that the strength of an ice cover usually is greater with bending from above than with bending from below (Neronov, 1946; Butiagin, 1955).

The higher the temperature of the ice, the smaller its fracture strength. According to the experimental data of F. F. Orlov (1940), the ultimate strength of ice decreases approximately 46% with an increase of temperature from -10°C to -0.5°C.

The shearing strength of ice. Table 11 presents the basic data on the magnitude of the shearing strength of ice, based on the information of various investigators. Obviously, the shearing strength may vary within broad limits. For the most part, the shearing strength is less than the tensile strength (Veinberg, 1940), on an average about half the tensile strength ($\sigma_{\text{tensile}} \approx 11.1\ \text{kg/cm}^2$, $\sigma_{\text{shear}} \approx 5.8\ \text{kg/cm}^2$). However, at low temperatures in individual cases the shearing strength may be considerably greater than the tensile strength. The magnitude of the shearing strength, as in the case of other types of destruction, increases with decreasing temperature and may change as a function of the ice structure and the direction of shear with respect to the direction of the crystal axes. Furthermore, as the experiments of Vialov (1958) have shown, the shear strength of ice is a function of the magnitude of the normal pressure in the shear plane, increasing as the pressure increases. There is also some basis for assuming that the shearing conditions exert a considerable influence on the magnitude of the shearing strength, viz., the manner of conducting the experiment, the size of the sample, the rate of application of the load or the rate of shear, et al. However, not enough study has been devoted to these problems.

The adfreezing (freezing together) strength of ice and various substances is a function of the material, the character of its surface and the temperature. Table 12 shows some values of the maximum adfreezing forces of ice. From the data it is evident that the adfreezing forces increase substantially with decreasing temperature and with roughness of the surface. The adfreezing forces also change as a function of the conditions whereby ice freezes together with another body and these are responsible for the structure of the

ice and the direction of the crystal axes at the point of contact. The adfreezing forces depend on the rate of increase of load. When the load increases rapidly, brittle fracture results and rupture may pass in part along the ice and not strictly along the contacts, depending on the material and the condition of its surface. For example, in experiments on the extraction of wooden rods (stakes) frozen into ice, Vialov (1956) observed cases where the destruction was accompanied by a sharp cracking sound and rupture of the ice, individual pieces of which remained on the extracted rod. The greatest values of the adfreezing forces were noted in these cases. However, when there was a prolonged interaction of loads or when the loads were increased slowly, the rod slipped along the ice. In such cases, the adfreezing forces were considerably smaller.

Figure 26 shows a curve of long-term adfreezing strength of ice with wooden rods (stakes) frozen into it (Vialov, 1956), which shows how the adfreezing strength varies as a function of the time of active loading up to the moment the stakes were extracted. When the load was increased rapidly, the adfreezing forces reached 5 kg/cm^2. With loads that created adhesive forces of 1 kg/cm^2, the rods were pulled out in 8-12 hours, while 1,000-3,000 hours elapsed before extraction of the rods with adhesive forces of 0.5 kg/cm^2.

<u>The resistance of ice to local crumpling</u> may be considerably greater than the resistance of ice to crushing. Korzhavin's (1955) data show that the ultimate local crumpling strength of ice may be 2-2.5 times greater than the ultimate strength in general unilateral compression. He proposes the following formula for cases of the crumpling of river ice

$$\sigma_{\text{crumpling}} = \sigma_{\text{compression}} \sqrt[3]{\frac{B}{b}} \qquad (32)$$

This formula defines the ultimate crumpling strength of ice $\sigma_{\text{crumpling}}$ as a function of the ultimate compressive strength $\sigma_{\text{compressive}}$, the width of the floe B and the width of the crumple area b (we have in mind the crumpling at the edge of a floe, along its entire vertical face).

When a solid body (a die) is inserted into ice, the magnitude of the resistance of the ice and the nature of its destruction are functions of the size

and shape of the die, the rate of insertion (or the rate of application of load to the die) and also the size and shape of the volume of ice into which the die is inserted. Finally, other factors which affect the compressive strength of the ice, namely, temperature, ice structure, et al., also play a role here. For example, Korzhavin (1955) observed that the force required to insert the die decreases 1.5 to 2-fold if a triangular die with a peak angle of 60° is used instead of a semi-circular die, other conditions being equal.

In the case of a slow increase of load and prolonged loading, causing stresses beneath the die which are small compared with the ultimate strength, the die penetrates into the ice smoothly due to the creep of the ice. A rapid increase of load causes brittle fracture of the ice with crack formation in a zone near the die.

The strength of river ice decreases considerably during the breakup period (1.5 to 3-fold). The sun's rays and heat cause the ice to begin to melt throughout its volume by the time of the spring breakup. First, melting occurs at the contacts between the crystals, where films of mineralized water formed during the freezing process, freezing and subsequently melting at a low temperature. During the melting of these interlayers, voids having a lower pressure formed and water could penetrate into them. As a result, the ice became cloudy and friable and became rapidly weaker.

To determine the possible forces of interaction between bridge supports or various hydrotechnical structures and ice during the spring breakup period, one may use the calculated values of ice strength as a function of the rate of movement of the ice (table 13) proposed by Korzhavin (1955).

The resistance of ice to dynamic loads. Usually the term dynamic loading is used to define loadings during which there is a substantial acceleration of the particles of the loaded body or of another body in contact with it, for example, in the case of impact or oscillations.

During forced oscillations, causing stresses which vary in sign, cracks may appear in the ice, gradually grow and cause destruction. Consequently, ultimate strength decreases with an increasing number of cycles of changing stresses.

B. D. Kartashkin (1947) noted that in most cases ice beams under a relatively small static load disintegrated during forced oscillations. The greatest additional dynamic stress which an ice beam was able to withstand for a fairly long time (10,000,000 cycles) without disintegrating, at a temperature of -5°C to -9°C under a static load causing a maximum stress of 2.5 kg/cm^2 with an ultimate strength of 16 kg/cm^2, was approximately 1.5 kg/cm^2. The smallest additional dynamic stress above which the beams disintegrated almost instantaneously was approximately 2.75 kg/cm^2. Thus, the ultimate strength decreased 3 to 4-fold during dynamic loading.

The nature of the impact deformation is a function of the active rate of loading (the impact). A small impact velocity causes only elastic deformations. When the rate of impact is increased, elasto-plastic deformations appear and finally brittle fracture. As yet, too little study has been devoted to problems of the resistance of ice to impact stresses.

TABLE I

Elastic Modulus of Ice, According to Data from the Static Method

Investigator	Type of Ice	Type of Loading	Temp. -°C	Elastic modulus x 10^3 kg/cm^2	Lit.
Bevan, 1824	Lake	Bending	---	52	Veinberg, 1940
Fabian, 1877	Artificial	Tension	0	17	"
Koch, 1833	Lake	Bending	---	70-90	"
Trowbridge and McRea, 1885	Pond	Bending	1	41-57	"
Ditto	"	"	3	58-72	"
Ditto	"	"	5	88-104	"
Ditto	"	"	7	59-83	"
Hess, 1902	Glacier	"	0-5	5-42	Hess, 1902
Koch, 1913	Lake	"	6-8	59-68	Koch, 1913
Koch, 1914	River	"	0	86-117	Koch, 1914
Matsuyama, 1920	"	", o ⊥ f and ℓ*	3.9	9	Lin'kov, 1957
Ditto	"	", o ∥ f	2.6	6	"
Ditto	"	", o ∥ ℓ	3.7	19	"
Pinegin, 1923	"	Bending	5.9	12	Pinegin, 1923
Ditto	"	"	15.19	21	"
Pinegin, 1922-1925	"	Compression, o ⊥ f	3	3-37	Pinegin, 1927
Ditto	"	Compression, o ⊥ f	5	48-84	"
Sokolov, 1926	Monocrystal	Bending	6	27	Sokolov, 1926
Ivanov, K. E.	-	Bending of ice cover (sheet)	---	44	Ivanov, 1946
Shul'man	-	-	---	--	Shul'man, 1948
Kobeko, 1946	-	-	---	--	Kobeko, 1946
Kartashkin, 1943-1945	Fluid (loose, pourable)	Compression	3.5	31	Kartashkin, 1947
Ditto	"	"	7-8	48-60	"
Ditto	"	Tension	9	36-55	"
Ditto	"	"	18	42-60	"
Ditto	Reservoir	"	6-7	25-46	"
Ditto	River	"	5.5-8	37-50	"
Ditto	River	Bending	1.5-21	35-62	
Ditto	Fluid (loose, pourable)	Bending	1-18.5	31-59	
Ditto	"	Bending	20-27	35-75	
Ditto	"		40	73-89	

* o is the direction of the optic axis of the crystals, f the direction of the force and ℓ the length of the sample.

TABLE I (CONT'D)

Investigator	Type of ice	Type of Loading	Temp -°C	Elastic Modulus x 10^3 kg/cm^2	Lit
Voitkovskii, 1954-1958	Artificial	"	1-4	25-65	-
Jellinek and Brill, 1956	Fine-grained	Tension	5-15	21-78	Jellinek and Brill, 1956
Ditto	Monocrystal	Tension, axis at 45° angle to force	5	49-83	"

TABLE II

The Elastic Modulus of Ice, According to Data of the Dynamic Method

Investigator	Type of ice	Method of Investigation	Temp. -°C	Elastic modulus x 10 kg/cm²	Lit.
Trowbridge and McRae, 1885	Artificial	Longitudinal and transverse prism oscillations	---	61-86	Veinberg, 1940
Brockamp and Mothes, 1929	Alpine-glaciers	Seismometric	---	69	Brockamp and Mothes, 1930
Boyle and Sproule, 1931	Artificial	Acoustic	9-20	90-94	Boyle and Sproule, 1931
Ditto	"	"	30-35	95-109	"
Ewing, Crary and Thorne, 1934	Artificial and Lake	"	5-15	88-98	Ewing, Crary and Thorne, 1934
Berdennikov, 1948	Artificial	"	2-40	88-97	Berdennikov, 1948
Nakaya, 1958	Glacier, density 0.914	"	9	90	Nakaya, 1958
Ditto	Glacier, density 0.90	"	9	70	"
Ditto	Glacier, density 0.70	"	9	40	"

TABLE III

Variation of the Elastic Modulus of Ice During Bending as a Function of Load

Range of change of load, in kg	Temperature, - °C	Beam 1		Beam 2		Beam 3	
		E, x10³ K2/cm²	Δδ, x10⁻³ cm	E, x10³ K2/cm²	Δδ, x10³ cm	E, x10³ K2/cm²	Δδ, x10⁻³ cm
0-12	3.0	53	5	65	5	65	5
12-0	3.2	65	4	65	5	53	4
0-21	2.9	66	7	57	8	57	8
21-0	2.7	51	9	51	9	46	10
0-30	3.0	51	13	37	18	35	19
30-0	3.1	55	12	32	21	--	--
0-40	2.1	38	23	37	24	44	20
40-0	1.8	33	27	34	26	--	--
0-40	3.9	42	21	40	22	29	30
40-0	3.2	37	24	37	24	40	22

TABLE IV

Variation of the Elastic Modulus with Repeated Loadings and Unloadings (Kartashkin, 1947)

Test Type	Temperature, - °C	Elastic modulus E x 10³ kg/cm²				
		E_1*	E_2	E_3	E_5	E_{20}
Compression	3.5	31.5	42.5	--	--	--
Tension	6.9	39.9	48.2	49.1	--	--
"	6	37.4	43.4	41.0	44.0	--
"	6	40.7	--	--	49.2	53.3
Bending	8	39.4	43.4	47.6	--	--
"	6.5	43.2	--	--	45.4	49.0
"	13.5	40.2	45.5	46.4	--	--

* The subscript with the E indicates the number of loadings for which the value of the elastic modulus was determined.

TABLE V

The Shear Modulus of Ice

Investigator	Type of ice	Method of Investigation	Temp. -°C	Shear modulus $x10^3$ kg/cm^2	Lit.
Veinberg, 1905	River	Torsion of cylinder, $^{o}\perp f$	0	10	Veinberg, 1906
Ditto	"	"	5	16	"
Ditto	Glacier	Torsion of cylinder	0	8	"
Ditto	"	"	5	34	"
Koch, 1914	Lake	Torsion of prism	---	28-30	Koch, 1914
Matsuyama, 1920	River	Torsion of cylinder, $^{o}\perp f$	7	2	Veinberg, 1940
Brockamp and Mothes, 1930	Glacier	Seismometer	ca. 0	25	Brockamp and Mothes 1930
Ewing, Crary and Thorne, 1934	Artificial	Torsional vibrations	5-15	34	Ewing, Crary and Thorne, 1934
Kartashkin, 1943-1945	Fluid (loose, pourable)	Torsion of a cylinder	4-5	10-11	Kartashkin, 1948
Ditto	"	"	11-16	10-21	"
Ditto	River	"	10-16	13-18	"
Voitkovskii, 1958	Artificial, random structure	Torsion of tubes of ice	4	12-18	-

TABLE VI

Steady-State Angular Strain Rates for Ice $\dot{\gamma}_\infty$ (10^{-6} 1/hr) With Normal and Shear Stresses Acting Simultaneously (for Tube 1*) at a Temperature of -1.2°C, for the Remaining Tubes, -4°C)

Stresses in kg/cm^2			Tube Number **									
			1			2	3	4	5	6		
τ	σ	S*	γ_∞	R_{actual}	$R_{theoretical}$	$\dot{\gamma}_\infty$	$\dot{\gamma}_\infty$	$\dot{\gamma}_\infty$	$\dot{\gamma}_\infty$	$\dot{\gamma}_\infty$	R_{actual}	$R_{theoretical}$
0.75	0	0.75	12	1.0	1.0	--	--	--	--	--	--	--
0.75	1.0	0.9	15	1.2	1.2	--	--	--	--	--	--	--
0.75	1.5	1.1	17	1.4	1.5	--	--	--	--	--	--	--
0.75	2.0	1.4	22	1.8	1.9	--	--	--	--	--	--	--
1.0	0	0	22	1.0	1.0	--	8	--	7	6	1.0	1.0
1.0	1.5	1.3	26	1.2	1.3	--	--	--	--	--	--	--
1.0	2.0	1.5	31	1.4	1.5	--	14	--	--	--	--	--
1.0	3.0	2.0	--	--	--	--	--	--	13	13	2.1	2.0
1.0	5.0	3.0	--	--	--	--	--	--	--	19	3.1	3.0
1.5	0	1.5	--	--	--	11	17	10	16	12	1.0	1.0
1.5	1.0	1.6	--	--	--	12	--	--	--	--	--	--
1.5	2.0	1.9	--	--	--	14	23	12	--	--	--	--
1.5	3.0	2.3	--	--	--	--	--	--	24	18	1.5	1.5
1.5	5.0	3.3	--	--	--	--	--	--	--	26	2.2	2.2
2.0	0	2.0	--	--	--	20	--	--	28	20	1.0	1.0
2.0	2.0	2.3	--	--	--	22	--	--	--	--	--	--
2.0	3.0	2.6	--	--	--	--	--	--	38	--	--	--
2.0	4.0	3.1	--	--	--	--	--	--	--	30	1.5	1.5
2.5	0	2.5	--	--	--	--	--	--	47	29	--	--
2.5	3.0	3.0	--	--	--	--	--	--	60	--	---	--
2.5	4.0	3.4	--	--	--	--	--	--	--	52**	--	--
3.0	0	3.0	--	--	--	--	--	--	85**	--	--	--

* According to formula (13)

$$S = \sqrt{\frac{\sigma^2}{3} + \tau^2}$$

for torsion and longitudinal compression of the tube.

** γ_∞ are the minimum shear rates in absence of the prolonged stage of steady creep; R_{actual} is the actual relative increase of the shear rate due to normal stresses

$$R_{actual} = \frac{\gamma}{\gamma(\sigma = 0)};$$

$R_{theoretical}$ is the same ratio calculated theoretically according to formula (16) when n = 2

$R_{theoretical} = S/\tau$

TABLE VII

The Ultimate Strength of Ice for Unilateral Compression

Investigator	Type of Ice	Temp. -°C	Strength, in kg/cm² $\sigma_{\perp}$	$\sigma_{\parallel}$	σ	Literature
Vasenko, 1899	Artificial	10-18	--	--	12-50	Vasenko, 1899
Ditto	River	12-17	20	37-46	--	"
Bell, 1911	River	0	37-55	25-54		Komarovskii
Ditto	River	8-10	--	--	34-78	"
Barnes and McKay, 1914	"	0	17-40	16-39	--	"
Bessonov, 1915	"	---	10-26	29-61	--	Bessonov, 1923
Sergeev, 1921	"	---	--	--	10-75	Sergeev, 1929
Pinegin, 1923	River, upper part	0-2	18	21	--	Pinegin, 1923
Ditto	"	12-15	25	29	--	"
Ditto	"	31-35	28	38	--	"
Ditto	River, middle part	0-2	28	36	--	"
Ditto	"	12-15	33	33	--	"
Ditto	"	31-35	69	76	--	"
Ditto	River, lower part	0- 2	12	18	--	"
Ditto	"	12-15	18	20	--	"
Ditto	"	31-35	32	38	--	"
Arnol'd-Alia-b'ev, 1923-28	From Gulf of Finland	0	22	--	--	Arnol'd-Alia-b'ev, 1929
Ditto	"	1	26	--	--	"
Ditto	"	2	35	--	--	"
Ditto	"	5	47	--	--	"
Ditto	"	9	56	--	--	"
Ditto	"	13	52	--	--	"
Korzhavin, 1934	River	0	10	30	--	Korzhavin, 1951
Ditto	"	0.3	12	22	--	"
Ditto	"	0.6	10	37	--	"
KOVM team, 1936-1937	Underground	0.1-9	--	--	9-32	Khomichev-skaia, 1940
Ditto	Ditto, contaminated with ground	3-7	--	--	16-43	"
Ditto	Ice of ground naled (icing)	0-86	--	--	16-122	"
Veinberg	Mean value for upper part	3	29	33	--	Veinberg, 1940
Ditto	Ditto, for lower part	3	23	28	--	"

TABLE VII (CONT'D)

Investigator	Type of Ice	Temp. -°C	Strength in kg/cm² $\sigma_{\perp}$	$\sigma_{\parallel}$	σ	Literature
Kartashkin 1943-1945	Reservoir	2	--	36-50	--	Kartashkin, 1947
Ditto	Fluid (loose pourable)	3.5	--	--	48-51	"
Ditto	"	8	--	--	48-83	"
Korzhavin, 1938	River	0	15	--	--	Korzhavin, 1952
Ditto	"	3.6	27	--	--	"
Korzhavin, 1950	"	0	10	--	--	"
Butiagin, 1953	"	0	5-19	--	--	Butiagin, 1955

Remarks: $\sigma_{\perp}$ is the ultimate compressive strength in a direction perpendicular to the crystal axes;

$\sigma_{\parallel}$, ditto, for compression in the direction of the crystal axes;

σ, ditto, in cases where there is no clearly defined crystal orientation in the test sample or where the orientation is not known.

TABLE VIII

The Ultimate Compressive Strength of Ice (kg/cm^2) as a Function of the Strain Rate and the Temperature

Strain rate v in cm/min*	Temperature, -°C					
	0	2	4	6	8	10
2	14.5	21.2	27.5	35.0	41.2	48.9
20	9.7	10.0	10.5	11.5	13.0	14.2

* The rate of compression of cubes $10 \times 10 \times 10$ cm^3 perpendicular to the crystal axes.

TABLE IX

Ultimate Tensile Strength of Ice

Investigator	Type of ice	Temp. -°C	Strength, kg/cm²	Lit.
Vasenko, 1897	Artificial	4-12	11-19	Vasenko, 1897
Hess, 1902	Glacier	-----	7	"
Pinegin, 1923	River, middle part, $^{o}\perp f$	0-2	10	Pinegin, 1923
Ditto	"	12-15	12	"
Ditto	"	31-35	14	"
Ditto	River, middle part, $^{o}\parallel f$	0-2	11	"
Ditto	"	12-15	15	"
Ditto	"	31-35	18	"
Ditto	River, lower part, $^{b}\perp f$	0-35	5-8	"
Ditto	Ditto, but $^{o}\parallel f$	0-35	10-13	"
Veinberg, 1940	Average value of 235 tests	-----	11.1	Veinberg, 1940
Kartashkin 1943-1945	River	3-8	9-12	Kartashkin 1947
Ditto	Fluid (loose, pourable)	3-18	10-18	"

TABLE X

Ultimate Bending Strength of Ice

Investigator	Type of ice	Temp. $-^{\circ}C$	Strength, kg/cm^2	Literature
Vasenko, 1897	River	15	25-45	Vasenko, 1897
Ditto	Artificial	15	30-42	"
Veinberg, 1912	River, upper pt.	6	8.3	Veinberg, 1913
Ditto	River, middle pt.	0	13.0	"
Ditto	River, lower pt.	0	12.7	"
Bessonov, 1913-1915	River	3	11-31	Bessonov, 1923
Sergeev, 1921	River, upper pt.	0	11.4	Sergeev, 1929
Ditto	River, middle pt.	0	9.9	"
Ditto	River, lower pt.	0	14.4	"
Pinegin, 1923	River	5-8	18	Pinegin, 1923
Ditto	"	15-19	33	
Pedder, 1925-28	"	0.2-32	5.7-22.1	Pedder, 1929
Basin, 1934	"	0	11.8	Korzhavin, 1951
Korzhavin, 1937	River, strain rate v=2 cm/min	0	9.2	"
Ditto	Ditto, v=20 cm/min	0	3.6	"
Orlov, 1940	River	8-10	3-45	Orlov, 1940
Ditto	River, fracture in water, v f	0	25	"
Ditto	River, fracture in water, ° f	0	14	"
Veinberg, 1940	River, av. value	3	16	Veinberg, 1940
Troshchinskii, 1942	River, flexure of ice strips in water	0	7.1	Korzhavin, 1951
Shishov, 1938-43	"	1	1.4-8.3 (av. 4)	Shishov, 1947
Ditto, 1942	River	2	9-13	"
Ditto	River	5	10-16	"
Ditto	"	10	13-20	"
Ditto	"	20	18-19	"
Neronov, 1943	River, flexure of ice strips*	0	2.8-5.6	Neronov, 1946

* Ice strips, literally pieces in the form of piano keys (tr. note)

TABLE X (CONT'D)

Investigator	Type of ice	Temp. -°C	Strength, kg/cm^2	Literature
Kartashkin, 1943-1945	River	3-21	8-24	Kartashkin, 1947
Ditto	Fluid (loose, pourable)	1-3	8-16	"
Ditto	"	4-27	12-23	"
Ditto	"	40	20-24	"
Butiagin, 1953	River, flexure of ice strips*	0	1.5-5.5 (av. 3.6)	Butiagin, 1955

* ice strips, literally pieces in the form of piano keys (tr. note)

TABLE XI

Ultimate Shearing Strength of Ice

Investigator	Type of ice	Temp. -°C	Strength, kg/cm^2	Literature
Pinegin, 1922-23	River, middle part, $^\circ\perp f$	0-2	6	Pinegin, 1923
Ditto	"	12-15	10	"
Ditto	"	31-35	13	"
Ditto	River, middle part, $^\circ\parallel f$	0-2	6	"
Ditto	"	12-15	9	"
Ditto	"	31-35	12	"
Ditto	River, lower part, $^\circ\perp f$	0-23	7-9	"
Ditto	" , $^\circ\parallel f$	0-23	6-9	"
Finlayson, 1927	River	1-24	5-35	Komarovskii, 1932
Sheikov and Tsytovich	Artificial	0	9	Tsytovich and Sumgin, 1937
Ditto	"	0.4	11	"
Ditto	"	2.9-6.1	27-38	"
Ditto	"	10.1	56	"
Veinberg, 1940	Av. value for 111 tests	----	5.8	Veinberg, 1940
Butiagin, 1956-1957	River (section of a strip between holes in the ice)	0	1.6-8.3 (av. 3.5)	Butiagin, 1958
Ditto	Ditto, before breakup of ice	0	2.2	"

TABLE XII

The Adhesion Between Ice and Other Substances

Investigator	Material and type of Surface	Temp. -°C	Adhesion force, kg/cm²	Literature
Bell, 1911	Concrete with plastered surface (Non-ionized)	0	8-11	Komarovskii, 1932
		1.1	13-16	"
Tsytovich, 1930	Wood (pine) with a smooth surface	1	5.2	Tsytovich and Sumgin, 1937
Ditto	"	5	6.2	"
Ditto	"	7	11.6	"
Ditto	"	10	13.7	"
Ditto	"	20	22.0	"
Ditto	"	5-10	11.5	"
Ditto	Concrete with a smooth surface	5-10	9.8	"
Al'tberg, 1948	Iron	0.085	0.14	Al'tberg, 1948
Ditto	"	0.32	0.52	"
Ditto	"	0.50	0.81	"
Ditto	"	1.09	2.95	"
Ditto	Asphalt (bitumen)	0.08	0.025	"
Ditto	"	1.09	0.28	"

TABLE XIII

Ultimate Strength of River Ice (kg/cm^2) During the Period of The Spring Break-up as a Function of the Rate of Ice Movement (m/sec)

Type of force	Rivers of the North and Siberia			Rivers of the European USSR		
	Ice motion	Full break-up		Ice motion	Full-break-up	
	0.5	1.0	1.5	0.5	1.0	1.5
Compression	6.5	5.0	4.5	3.5	2.5	2.0
Local crumpling	16.0	13.0	11.5	8.0	6.5	5.5
Bending	7.5	6.0	5.5	4.0	3.5	3.0
Shear	--	3-6	--	--	1.5-3.0	--
Tension	--	7-9	--	--	3-4	--

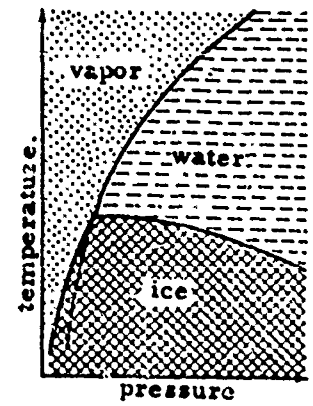

Fig. 1. Schematic diagram of the phase state of water (the dashed line is the equilibrium curve of vapor and supercooled water).

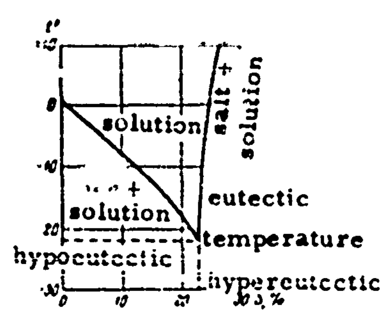

Fig. 2. Curves of the solidification of an aqueous solution of sodium chloride.

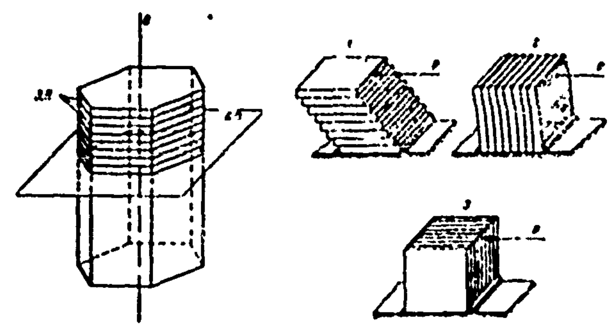

Fig. 3. Basic directions of the shearing forces with respect to the basal plane of the crystal.

O-optic axis of the crystal; BP-basal plane; EP-elementary plates; P-shearing force; 1-direction of the shear plane coincides with BP; 2-direction of the shearing force and shear plane is perpendicular to the BP; 3-direction of the shearing force coincides with BP, but the shear plane is perpendicular to the BP.

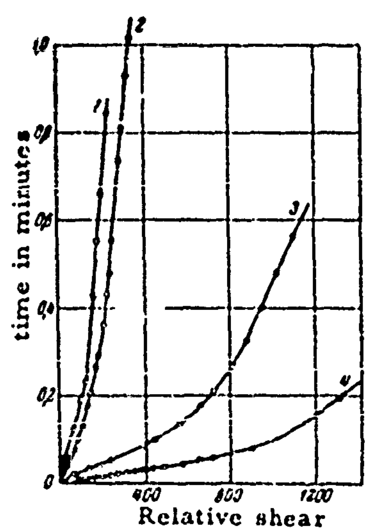

Fig. 4. Shear strain of ice monocrystals at temperature -2.3°C, when the shear plane coincides with the basal plane.
1 - τ = 2.2 kg/cm^2; 2 - τ = 1.9 kg/cm^2; 3 - τ = 0.55 kg/cm^2
4 - τ = 0.45 kg/cm^2.

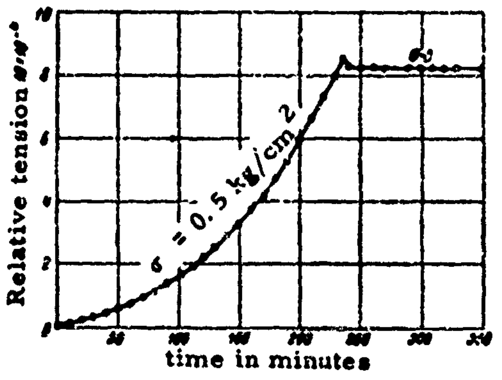

Fig. 5. Tensile strain of an ice monocrystal at temperature $-5^{\circ}C$ (tension at an angle of 45° to the crystal axis).

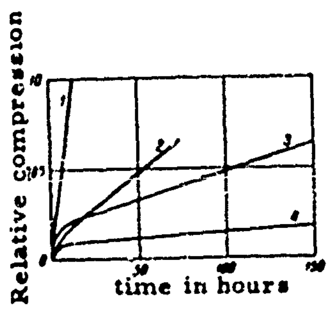

Fig. 6. Curves of the compressive strain of polycrystalline ice samples, after Glen (1955)

1. $\sigma = 6.1$ kg/cm^2, t = $-0.02^{\circ}C$; 2. $\sigma = 3.6$ kg/cm^2, t = $-0.02^{\circ}C$; 3. $\sigma_2 = 6.1$ kg/cm^2, t = $-6.7^{\circ}C$; 4. $\sigma = 6.0$ kg/cm^2, t = $-12.7^{\circ}C$.

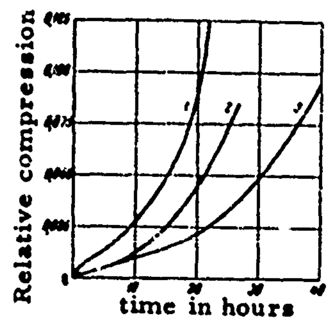

Fig. 7. Curves of the compressive strain of ice.
1. σ = 21 kg/cm^2; t = -6.7°C; 2. σ = 16 kg/cm^2; t = -5°C; 3. σ = 15 kg/cm^2; t = -6.4°C

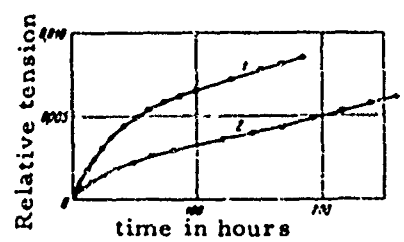

Fig. 8. Curves of the tensile strain of ice.

1. σ = 5.3 kg/cm^2, t = -7°C; 2. σ = 4.75 kg/cm^2, t = -7°C.

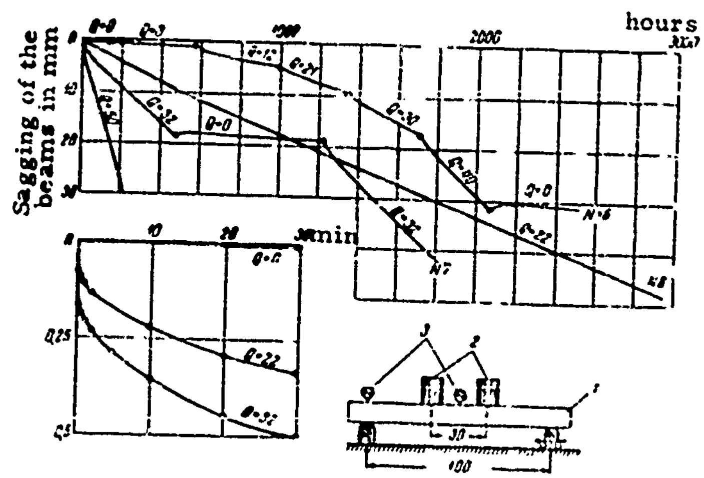

Fig. 9. The sagging of ice beams at a temperature of about -2°C and various loads Q in kg.

1. ice beam; 2. loads (weights); 3. indicators for determining the displacements of the beam.

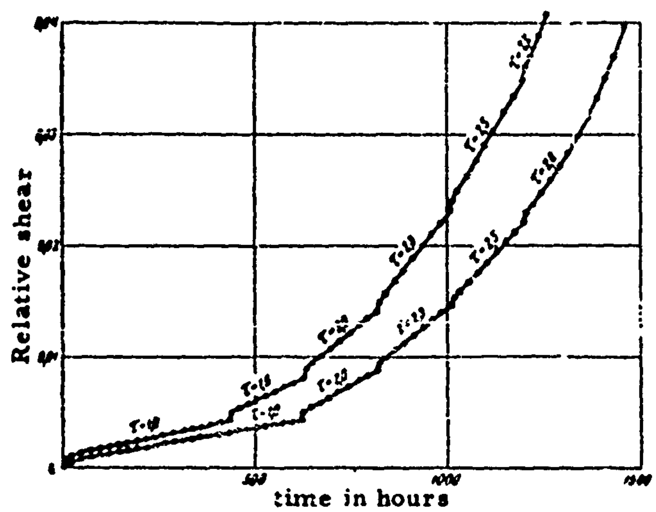

Fig. 10. Pure shearing of ice at a temperature of -4°C
(τ is tangential stress in kg/cm^2)

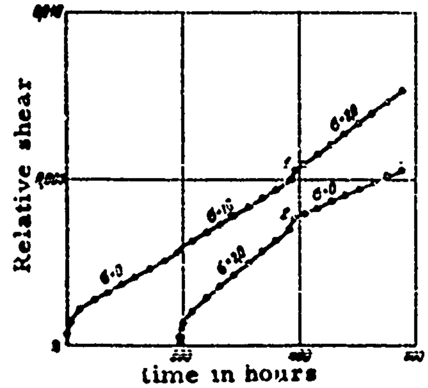

Fig. 11. Shear strain with simultaneous tangential and normal stresses in the author's experiments to determine the torsion and longitudinal compression of ice tubes:

τ - tangential stress; σ - normal stress; 1. τ = 1.5 kg/cm^2; 2. τ = 1.0 kg/cm^2.

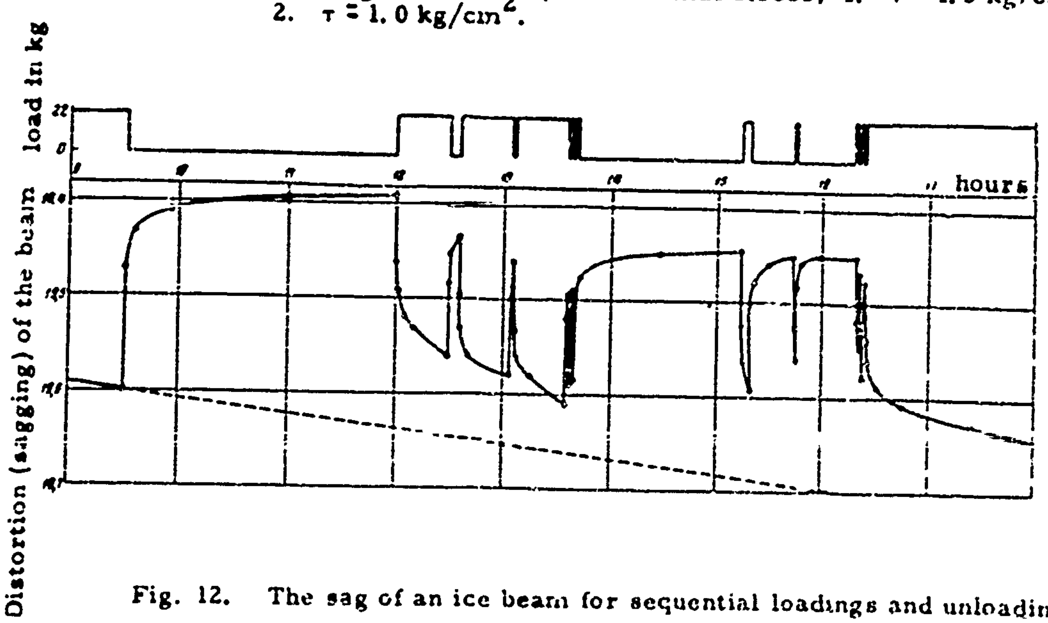

Fig. 12. The sag of an ice beam for sequential loadings and unloadings.

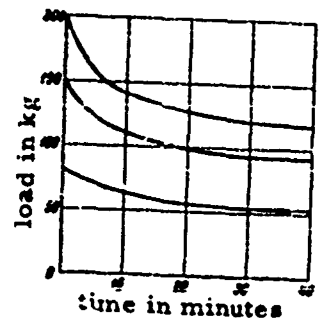

Fig. 13. Curves of relaxation

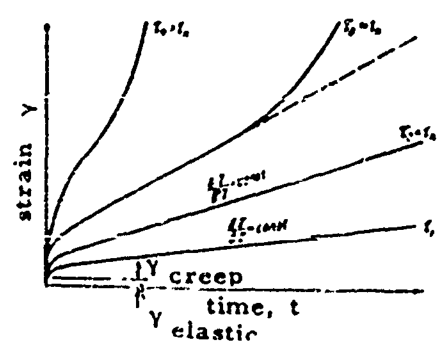

Fig. 14. Curves of ice creep (τ_1 = limit of prolonged creep)

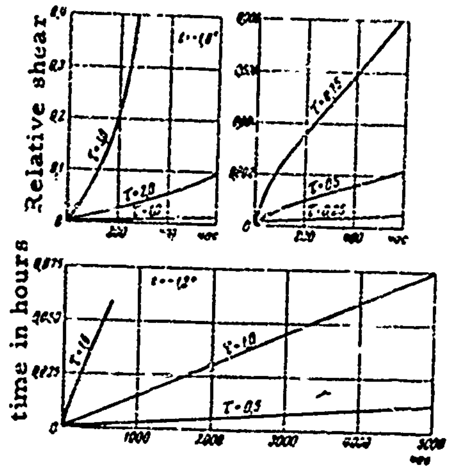

Fig. 15. Curves of the creep of polycrystalline ice in pure shear at temperatures -1.2°C and -1.8°C (τ is the tangential stress in kg/cm^2)

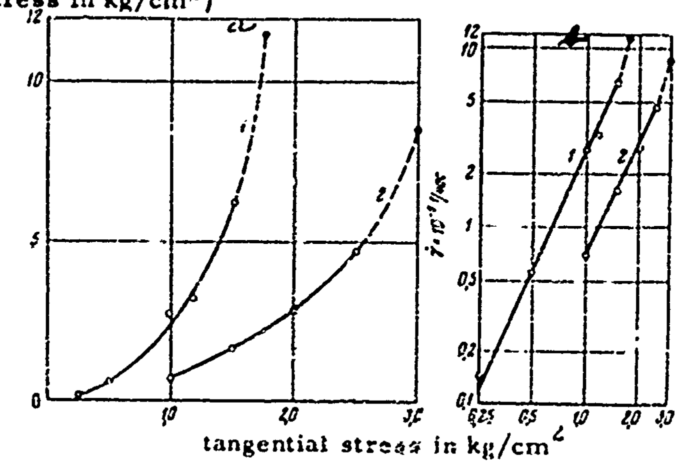

Fig. 16. Change in the steady state shear as a function of tangential stress: a-graph in ordinary coordinates; b-the same, in logarithmic coordinates; 1. at t = -1.2°C; 2. at t = -4°C; the black dots indicate points corresponding to the minimum shear velocity with stresses exceeding the limit of prolonged creep, when there is no prolonged stage of steady creep.

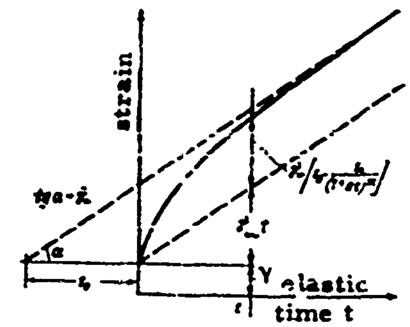

Fig. 17. **Diagram of the arbitrary division of the total strain, according to formula (8).**

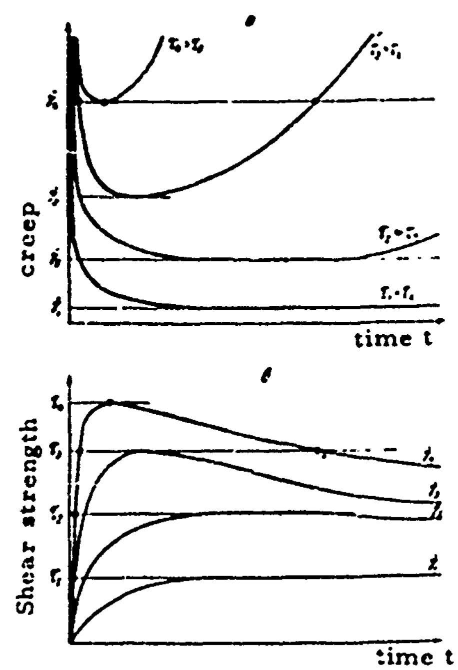

Fig. 18. Diagrams of the change of the ice creep rate with a permanent stress (a) and the change in the resistance with steady shear rates (b).

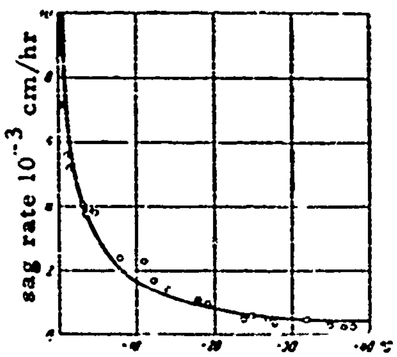

Fig. 19. Rate of sag of an ice beam as a function of temperature (under its own weight and an additional weight Q = 40 kg)

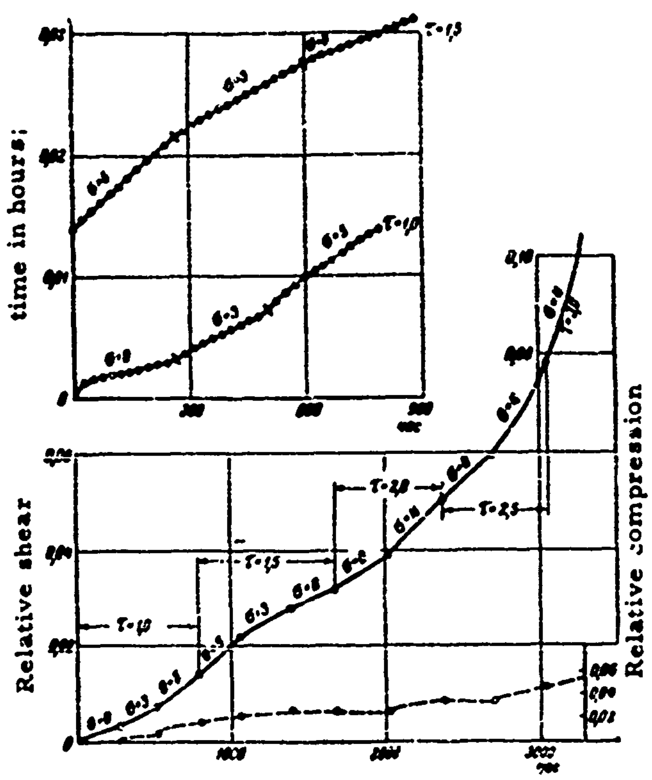

Fig. 20. Creep of ice with normal and tangential stress acting simultaneously at temperature -3.8°C.

Solid line - change of relative shear; dashed line: change of relative compression; σ normal stress in kg/cm^2; τ tangential stress in kg/cm^2.

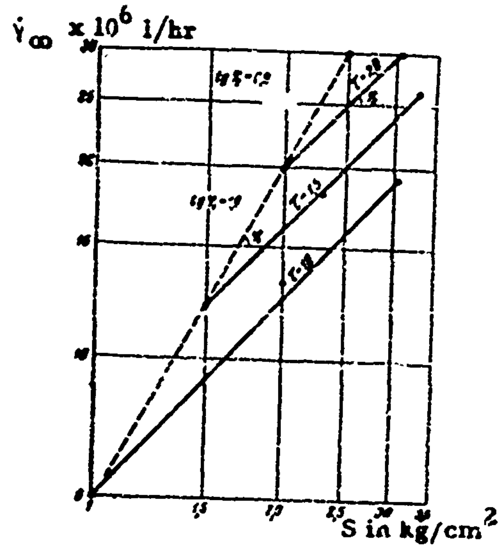

Fig. 21. Steady creep rate with simultaneous shearing and compression at -3.8°C.

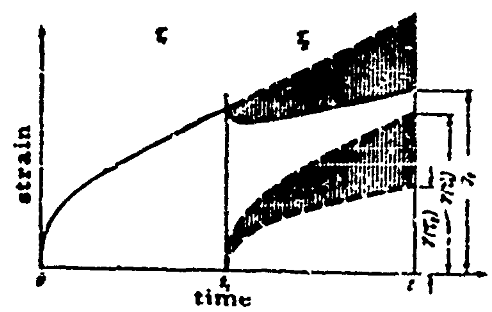

Fig. 22. Diag. -m of the computed value of ice strain with a change in stres : $\tau_2 < \tau_1$.

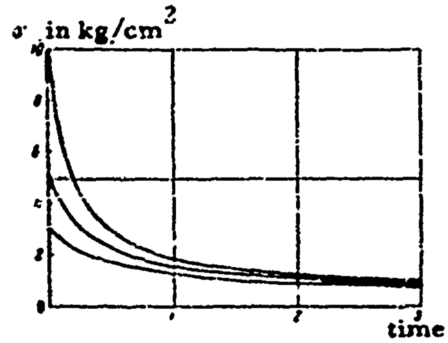

Fig. 23 Theoretical curves of the relaxation of stresses during unilateral compression, calculated on the basis of formula (29).

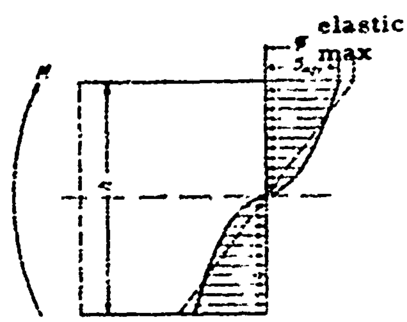

Fig. 24. The distribution of stresses in a cross section of a flexed ice beam.

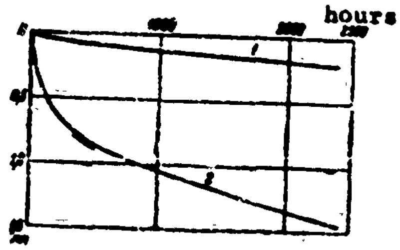

Fig. 25. Indentation of a flat, round die 30 cm^2 into ice at $t = -3.5°C$.

1. with $\tau = 2$ kg/cm^2; 2. with $\sigma = 5$ kg/cm^2

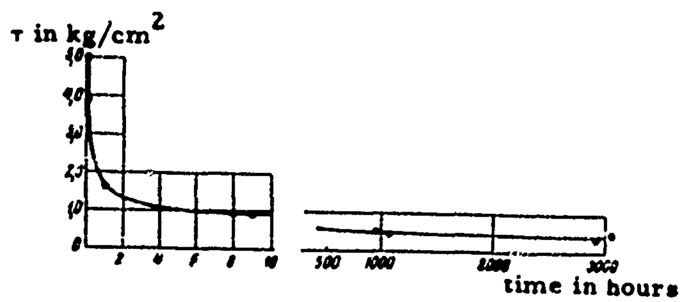

Fig. 26. Curve of long-term strength of freezing of ice to wood at $t = -0.4°C$.

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